## VIVEKANANDA COLLEGE, TIRUVEDAKAM WEST

Residential \& Autonomous - A Gurukula Institute of Life-Training Re-accredited ( ${ }^{\text {rd }}$ Cycle) with 'A' Grade (CGPA 3.59 out of 4.00 ) by NAAC
[Affiliated to Madurai Kamaraj University]
B.Sc. Physics/Chemistry Degree (Semester) Examinations, April 2022 Part - III: Allied Course: Fourth Semester: Paper - I

## MATHEMATICS - II

Under CBCS and LOCF - Credit 3

## SECTION - A

## Answer ALL Questions

$(10 \times 1=10)$

1. The order of the differential equation $\frac{d y}{d x}+y \cot x=0$ is $\qquad$
a) 1
b) 2
c) 4
d) 3
2. The degree of the differential equation $y^{\prime \prime \prime}=(x+y)^{\frac{7}{4}}$ is
a) 2
b) 4
c) 6
d) 3
3. The integrating factor of the differential equation $\frac{d y}{d x}+y \cot x=2 x \sin x$ is
a) $\operatorname{cosec} x$
b) $\sin x$
c) $\sec x$
d) $\cos x$
4. In an Exact differential equation, when $M d x+N d y \neq 0$ and the equation is homogeneous $\qquad$ is an integrating factor of $M d x+$ $N d y=0$
a) $\frac{1}{M x+N y}$
b) $\frac{-1}{M x+N y}$
c) $\frac{1}{M x-N y}$
d) $\frac{-1}{M x-N y}$
5. The auxiliary equation of $x^{2} \frac{d^{2} y}{d x^{2}}+3 x \frac{d y}{d x}+y=\frac{1}{(1-x)^{2}}$ is
a) $(m+1)^{2}=0$
b) $(m-1)^{2}=0$
c) $(m+2)^{2}=0$
d) $(m-2)^{2}=0$.
6. The Particular integral of the differential equation $\left(D^{2}-9\right) y=\cos 3 x$ is
a) $\frac{\cos 3 x}{18}$
b) $\frac{\cos 3 x}{-18}$
c) $\frac{\cos 3 x}{-9}$
d) none of them
7. The Auxiliary equation of the differential equation $x^{2} \frac{d^{2} y}{d x^{2}}-3 x \frac{d y}{d x}-$
$5 y=0$ is
$\qquad$
a) $m^{2}-4 m+5=0$
b) $m^{2}+4 m+5=0$
c) $m^{2}-4 m-5=0$
d) $m^{2}-4 m=-5$
8. The particular integral of the differential equation $y^{\prime \prime \prime}-y=0$ is
a) 1
b) 2
c) 3
d) 0
9. One of the solutions of the differential equation $\frac{d x}{3}=\frac{d y}{y}=\frac{d z}{3}$ is $\qquad$
10. a) Show that $x d x+y d y-\left(\frac{x d y-y d x}{x^{2}+y^{2}}\right)=0$ is exact and solve it.
[OR]
b) Show that $\left(x^{2}+y^{2}+x\right) d x+x y d y=0$ is not exact and solve it.
11. a) Solve $\left(D^{2}-4\right) y=e^{3 x}+e^{-4 x}$

## [OR]

b) Show that the particular integral of the differential equation $\left(D^{2}+9\right) y=4 \sin 3 x$
21. a) Solve $x^{2} y^{\prime \prime}+3 x y^{\prime}+y=\frac{1}{(1-x)^{2}}$

## [OR]

b) Find the solution. Given that $\mathrm{y}=\mathrm{x}$ is a particular solution of the differential equation $x^{2} y^{\prime \prime}-2 x(1+x) y^{\prime}+2(1+x) y=x^{3}$
22. a) Show that the condition of integrability is satisfied for the equation $(2 x z-y z) d x+(2 y z-z x) d y-\left(y^{2}-z x+y^{2}\right) d z=0$.
[OR]
b) Determine the solution: $\frac{x d x}{y^{2} x}=\frac{d y}{x z}=\frac{d z}{y^{2}}$

## SECTION - D

## Answer any THREE Questions

23. Solve $\frac{d y}{d x}=\frac{6 x-4 y+3}{3 x-2 y+1}$
24. Solve $\left(2 x y^{4} e^{y}+2 x y^{3}+y\right) d x+\left(x^{2} y^{4} e^{y}-x^{2} y^{2}-3 x\right) d y=0$
25. Solve $\left(D^{2}-4 D+4\right) Y=3 x^{2} e^{2 x} \sin 2 x$
26. Apply the method of variation of parameters to solve

$$
y^{\prime \prime}+3 y^{\prime}+2 y=x^{2}
$$

27. Solve $\frac{d x}{y^{2}(x-y)}=\frac{d y}{-x^{2}(x-y)}=\frac{d z}{z\left(x^{2}+y^{2}\right)}$

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B.Sc. Mathematics Degree (Semester) Examinations, April 2022 Part - III: Allied Course: Fourth Semester: Paper - I

## SECTION - A

## Answer ALL Questions

$(10 \times 1=10)$

1. The wrapping up of data and functions into a single unit is called $\qquad$
a) inheritance
b) polymorphism
c) encapsulation
d) data hiding
2. Execution of all $\mathrm{C}++$ programs begins at $\qquad$ function
a) \#include
b) main( )
c) return( )
d) member
3. $\mathrm{C}++$ provides an additional use of $\qquad$ for declaration of generic pointers.
a) int
b) float
c) void
d) double
4. Default values for a function are specified when $\qquad$
a) function is defined
b) function is declared
c) Both a and b
d) None of these
5. The binding of data and functions together into a single class-type variable is referred to as $\qquad$
a) encapsulation
b) data hiding
c) data abstraction
d) data binding
6. A static member function can be called using the $\qquad$
$\qquad$ instead of its objects.
a) variable name
b) function name
c) Class name
d) object name
7. State whether the following statements about the constructor are True or False.
i) constructors should be declared in the private section.
ii) constructors are invoked automatically when the objects are created.
a) True, True
b) True, False
c) False, True
d) False, False
8. We can overload almost all the $\mathrm{C}++$ operators except the following.
i) Class member operator (.,.*)
ii) Assignment operator (=)
iii) Scope resolution operator (::)
iv) Conditional operator (?:)
a) i, ii and iii only
b) ii, iii and iv only
c) i, iii and iv only
d) All i, ii, iii and iv
9. Which among the following best describes the Inheritance?
a) Copying the code already written
b) Using the code already written once
c) Using already defined functions in programming language
d) Using the data and functions into derived segment
10. Which symbol is used to create multiple inheritance?
a) Dot
b) Comma
c) Dollar
d) None of the mentioned

## $\underline{\text { SECTION - B }}$

## Answer any FIVE Questions

$$
(5 \times 2=10)
$$

11. Define Encapsulation.
12. Recall any four math library functions.
13. Write the general form of a class declaration.
14. What are parameterized constructors?
15. Define Inheritance.
16. What is an abstract class?
17. Mention the operators that cannot be overloaded.

## SECTION - C

## Answer ALL Questions

$(5 \times 5=25)$
18. a) Enumerate the benefits of OOPs.
[OR]
b) List any five applications of OOPs.
19. a) Elucidate how to call the function by reference.

## [OR]

b) Summarize on Inline functions with examples.
20. a) List the characteristics of friend functions.
[OR]
b) Recapitulate on array of objects.
21. a) Discuss about copy constructors.
[OR]
b) Differentiate between constructor and destructor.
22. a) Describe how to define derived classes in $\mathrm{C}++$.
[OR]
b) Outline multilevel inheritance with a sample program.

## $\underline{\text { SECTION - D }}$

## Answer any THREE Questions

$(3 \times 10=30)$
23. Explicate the Basic concepts of OOPs.
24. Examine function overloading with an example.
25. Elaborate on defining member functions inside and outside the class definition.
26. Describe how to overload binary operators with an example.
27. Classify the different types of inheritance.

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B.Sc. Mathematics Degree (Semester) Examinations, April 2022 Part - III: Core Course: Second Semester: Paper - I

INTEGRAL CALCULUS
Under CBCS and LOCF - Credit 4
Time: 3 Hours
Max. Marks: 75

## SECTION - A

## Answer ALL Questions <br> $(10 \times 1=10)$

1. $\int \sec ^{2} x d x=$ $\qquad$
a) $\tan x+c$
b) $-\tan x+c$
c) $\cot x+c$
d) none
2. $\int \frac{d x}{\sqrt{x^{2}-a^{2}}}=$ $\qquad$
a) $\cos h^{-1}\left(\frac{x}{a}\right)+c$
b) $\frac{1}{a} \cos h^{-1}\left(\frac{x}{a}\right)+c$
(c) $\frac{1}{2 a} \log \left(\frac{x-a}{x+a}\right)+c$
d) none
3. If $f(x)$ is an even function, then $\int_{-a}^{a} f(x) d x=$ $\qquad$
a) 0
b) $2 \int_{0}^{a} f(x) d x$
c) $\frac{1}{2} \int_{0}^{-a} f(x) d x$
d) none
4. If n is even, $\int_{0}^{\frac{\pi}{2}} \sin ^{n} x d x=$ $\qquad$
a) $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdot \ldots \cdot \frac{2}{3} \cdot 1$
b) $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \ldots \cdot \frac{1}{2} \cdot \frac{\pi}{2}$
c) $\frac{n-1}{n} \cdot \frac{n-3}{n-2} \ldots \cdot \frac{2}{3} \cdot 1 \cdot \frac{\pi}{2}$
d) none
5. The Cartesian limits for the integration over the upper half of the circle $x^{2}+y^{2}=a^{2}$ are
a) $0 \leq x \leq a \quad, 0 \leq y \leq \sqrt{a^{2}-x^{2}}$
b) $-a \leq x \leq a, 0 \leq y \leq \sqrt{a^{2}-x^{2}}$
c) $0 \leq x \leq a \quad,-\sqrt{a^{2}-x^{2}} \leq y \leq \sqrt{a^{2}-x^{2}}$
d) none
6. $\Gamma(n+1)=$ $\qquad$
a) $(n+1)$ !
b) $\frac{n(n+1)}{2}$
c) $n$ !
d) none
7. The area of the ellipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$ is
a) $\pi a b$
b) $\pi^{2} a b$
c) $\pi \sqrt{a b}$
d) none
8. If $x=u(1-v), y=u v(1-w), z=u v w$ then $d x d y d z=$
$\qquad$ dudvdw
a) $u v$
b) $u^{2} v$
c) $u v^{2}$
d) none
9. The Fourier coefficient $b_{\mathrm{n}}$ in the Fourier series for $f(x)$ in $[-\pi, \pi]$ is given by
a) 0
b) $\frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin n x d x$
c) $\frac{2}{\pi} \int_{0}^{\pi} f(x) \sin x d x$
d) none
10. The Fourier coefficient $a_{n}$ in the half range Fourier cosine series for $f(x)$ in $[0, \pi]$ is
a) 0
b) $\frac{1}{\pi} \int_{0}^{\pi} f(x) \cos n x d x$
c) $\frac{2}{\pi} \int_{0}^{\pi} f(x) \cos n x d x$
d) none

## $\underline{\text { SECTION - B }}$

## Answer any FIVE Questions

$(5 \times 2=10)$
11. Find the value of $\int \sin ^{2} 3 x d x$.
12. Find $\int x e^{x} d x$.
13. Find $\int_{0}^{\frac{\pi}{2}} \sin ^{6} x \cos ^{5} x d x$.
14. Find $\iint x y d x d y$ taken over the positive quadrant of the circle $x^{2}+$ $y^{2}=a^{2}$.
15. Evaluate $\int_{0}^{\infty} e^{-x} d x$.
16. If $u=x^{2}-y^{2}$ and $v=x^{2}+y^{2}$ then find $\frac{\partial(u, v)}{\partial(x, y)}$.
17. Define Fourier Series.

## SECTION - C

## Answer ALL Questions

$$
(5 \times 5=25)
$$

18. a) Evaluate $\int \frac{d x}{1+\tan x}$
[OR]
b) Evaluate $\int \frac{d x}{4 x^{2}-4 x+2}$
19. a) Evaluate $\int(\log x)^{2} d x$
[OR]
b) Evaluate $\int \tan ^{4} x d x$
20. a) Evaluate $\iint\left(x^{2}+y^{2}\right) d x d y$ over the region for which $x, y$ are each $\geq 0$ and $x+y \leq 1$

## [OR]

b) By transforming into polar co. Ordinates evaluate $\iint \frac{x^{2} y^{2}}{x^{2}+y^{2}} d x d y$ over the annular region between the circles $x^{2}+y^{2}=a^{2}$ and $x^{2}+y^{2}=b^{2}$ $(b>a)$.
21. a) Evaluate $\iiint x y z d x d y d z$ taken through the positive octant of the sphere $x^{2}+y^{2}+z^{2}=a^{2}$

## [OR]

b) Evaluate $\iint_{R}(x-y)^{4} e^{x+y} d x d y$ where R is the square with vertices $(1,0),(2,1),(1,2)$ and $(0,1)$.
22. a) Express $f(x)=\frac{1}{2}(\pi-x)$ as a Fourier series with period $2 \pi$ to be valid in the interval 0 to $2 \pi$

## [OR]

b) Find a sine series for $f(x)=c$ in the range 0 to $\pi$

## $\underline{\text { SECTION - D }}$

## Answer any THREE Ouestions

$(3 \times 10=30)$
23. Solve $\int \frac{3 x+1}{(x-1)^{2}(x+3)} d x$
24. Evaluate $\int \operatorname{cosec}^{n} x d x$ and hence find $\int \operatorname{cosec}^{4} x d x$
25. Solve $\int_{0}^{\frac{\pi}{2}} \sqrt{\tan \theta} d \theta$
26. Show that $\iint_{D} e^{-x^{2}-y^{2}} d x d y=\frac{\pi}{4}\left(1-e^{-R^{2}}\right)$ where D is the region $x \geq$

0, $y \geq 0$ and $x^{2}+y^{2} \leq R^{2}$
27. Find a Fourier series with period 3 to represent $f(x)=2 x-x^{3}$ in the range $(0,3)$.

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B.Sc. Mathematics Degree (Semester) Examinations, April 2022 Part - III: Core Course: Second Semester: Paper - II
ANALYTICAL GEOMETRY 3D AND VECTOR CALCULUS
Under CBCS and LOCF - Credit 4
Time: 3 Hours
Max. Marks: 75

## SECTION - A

## Answer ALL Questions

$(10 \times 1=10)$

1. The equation of the $x z-$ plane is $\qquad$
a) $x=0$
b) $y=0$
c) $Z=0$
d) $x=0=y$
2. The mid-point of the line joining of two points $\mathrm{P}(1,2,3)$ and $\mathrm{Q}(3,4,5)$ is
a) $(2,3,4)$
b) $(1,2,3)$
c) $(-1,2,3)$
d) $(2,-3,1)$
3. The image of the point $(1,2,3)$ under the reflection in the $y z$ - plane is
a) $(1,2,3)$
b) $(1,2,-3)$
c) $(1,-2,3)$
d) $(-1,2,3)$
4. The mirror reflection of the point $(1,2,3)$ in the $x z$ - plane is $\qquad$
a) $(1,2,-3)$
b) $(1,-2,3)$
c) $(-1,2,3)$
d) $(1,2,3)$
5. The equation of the sphere whose center is origin and radius 1 is $\qquad$
a) $x^{2}+y^{2}+z^{2}=9$
b) $x^{2}+y^{2}+z^{2}=2$
c) $x^{2}+y^{2}+z^{2}=4$
d) $x^{2}+y^{2}+z^{2}=1$
6. The xz -plane section of the sphere $x^{2}+y^{2}+z^{2}=a^{2}$ is $\qquad$ .
a) $y^{2}+z^{2}=a^{2}, x=0$
b) $x^{2}+z^{2}=a^{2}, y=0$
c) $x^{2}+y^{2}=a^{2}, z=0$
d) none
7. A vector function $\vec{f}$ is called irrotational if $\qquad$

## SECTION - C

a) $\operatorname{div} f=0$
b) $\operatorname{curl} \mathrm{f}=0$
c) $\operatorname{div} \vec{f}=0$
d) $\operatorname{curl} \vec{f}=0$
8. For the unit vectors $\vec{\imath} \times \vec{\imath}=\vec{\jmath} \times \vec{\jmath}=\vec{k} \times \vec{k}=$ $\qquad$
a) 0
b) 1
c) 2
d) 3
9. The value of $\iiint_{0}^{a} d x d y d z=$ $\qquad$
a) $a^{3}$
b) $a^{2}$
c) a
d) 1
10. Gauss divergence theorem connects
a) line integral and double integral
b) line integral and surface integral
c) double integral and surface integral
d) surface integral and volume integral

## SECTION - B

## Answer any FIVE Questions

11. Show that the points $(5,3,-2),(3,2,1)$ and $(-1,0,7)$ are collinear.
12. Find the equation of the plane through $(3,4,5)$ parallel to the plane $2 x+$ $3 y-z=0$.
13. Find the point where the line $\frac{x-2}{2}=\frac{y-4}{-3}=\frac{z+6}{4}$ meets the plane $2 \mathrm{x}+$ $4 y-z-2=0$
14. Find the coordinates of centre and radius of the sphere $x^{2}+y^{2}+z^{2}-2 x+6 y$ $+4 \mathrm{z}-35=0$.
15. Define irrotational.
16. Prove that $\nabla \circ \vec{r}=3$.
17. If $\vec{A}$ and $\vec{B}$ are irrotational, show that $\vec{A} \times \vec{B}$ is solenoidal.

## Answer ALL Questions

$(5 \times 5=25)$
18. a) Find the direction cosines of the line joining the points $(3,-5,4)$ and $(1,-8,-2)$

## [OR]

b) Find the angle between the planes $2 x-y+z=6$; $x+y+2 z=3$.
19. a) Find the image of the point $(1,-2,3)$ in the plane $2 x-3 y+2 z+3=0$.

## [OR]

b) Prove that the lines $\frac{x+1}{-3}=\frac{y+10}{8}=\frac{z-1}{2} ; \frac{x+3}{-4}=\frac{y+1}{7}=\frac{z-4}{1}$ are coplanar. Find also their point of contact.
20. a) Find the equation of the sphere through the four points and determine its radius $(0,0,0),(a, 0,0),(0, b, 0),(0,0, c)$.

## [OR]

b) Find the equation of the sphere which touches the sphere $x^{2}+y^{2}+z^{2}-$ $6 x+2 z+1=0$ at the point $(2,-2,1)$ and passes through the origin.
21. a) Find the directional derivative of $\phi(x, y, z)=x y^{2}-y z^{3}$ at the point ( 2 ,
$-1,1)$ in the direction of the vector $\vec{i}+2 \vec{j}+2 \vec{k}$.
[OR]
b) Prove that $\vec{F}=\left(y^{2} \cos x+z^{3}\right) \vec{i}+(2 y \sin x-4) \vec{j}+\left(3 x z^{2}+2\right) \vec{k}$ is irrotational and find its scalar potential.
22. a) Given the vector field $\vec{F}=x z \vec{i}+y z \vec{j}+z^{2} \vec{k}$, evaluate $\int_{C} \vec{F} o d \vec{r}$ from the point $(0,0,0)$ to $(1,1,1)$, where $C$ is the curve $x=t, y=t^{2}, z=t^{3}$.

## [OR]

b) Show that $\iint_{S} \vec{r} \circ \vec{n} d s=3 V$, where V is the volume enclosed by S and r is the position vector.

## $\underline{\text { SECTION - D }}$

## Answer any THREE Ouestions <br> $(3 \times 10=30)$

23. Find the equation of the plane passing through the points $(2,-5,3)$,
$(-2,-3,5)$ and $(5,3,-3)$.
24. Find the shortest distances between the lines

$$
\frac{x-3}{-1}=\frac{y-4}{2}=\frac{z+2}{1} ; \frac{x-1}{1}=\frac{y+7}{3}=\frac{z+2}{2} .
$$

25. Find the equation of the sphere which passes through the circle
$x^{2}+y^{2}+z^{2}-2 x-4 y=0, x+2 y+3 z=8$ and touches the plane $4 x+3 y=25$.
26. Prove that $\vec{v}=r^{n} \vec{r}$ is irrotational. Find n when it is also solenoidal.
27. Evaluate $\iint\left(x^{3} d y d z+x^{2} y d z d x+x^{2} z d x d y\right)$ over the surface bounded by $\mathrm{z}=0, \mathrm{z}=\mathrm{c}, \mathrm{x}^{2}+\mathrm{y}^{2}=\mathrm{a}^{2}$.

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B.Sc. Mathematics Degree (Semester) Examinations, April 2022 Part - III: Core Course: Fourth Semester: Paper - I

SEQUENCE AND SERIES
Under CBCS and LOCF - Credit 4
Time: $\mathbf{3}$ Hours
Max. Marks: 75

## $\underline{\text { SECTION - A }}$

## Answer ALL Questions

$(10 \times 1=10)$

1. Let $\mathrm{A}=(0,1)$, the glb and lub of A is $\qquad$
a) 1,0
b) 0,1
c) 0,0
d) 1,1
2. The first four terms of the sequence $\left(\frac{(-1)^{n+1}}{n!}\right)$ are $\qquad$
a) $-1, \frac{1}{2!},-\frac{1}{3!}, \frac{1}{4!}, \ldots$
b) $1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \ldots$
c) $-\frac{1}{2!}, \frac{1}{3!},-\frac{1}{4!}, \frac{1}{5!}, \ldots$
d) $1,-\frac{1}{2!}, \frac{1}{3!},-\frac{1}{4!}, \frac{1}{5!}, \ldots$
3. $\lim _{n \rightarrow \infty} n^{\frac{1}{n}}=$
a) 0
b) 2
c) -1
d) 1
4. $\lim _{n \rightarrow \infty} \frac{2 n+1}{2 n}$
a) 0
b) 1
c) 2
d) -1
5. $\qquad$ is the example of Cauchy sequence.
a) $\left(\frac{1}{n}\right)$
b) $(-1)^{\mathrm{n}}$
c) $n$
d) none
6. Ceasaro's theorem is $\qquad$
a) $\left(a_{n}\right) \rightarrow a \Rightarrow\left(\frac{a_{1}+a_{2}+\ldots \ldots \ldots+a_{n}}{n}\right) \rightarrow a$
b) $\left(a_{n}\right) \rightarrow a,\left(b_{n}\right) \rightarrow b \Rightarrow\left(\frac{a_{1} b_{n}+a_{2} b_{n-1}+\ldots \ldots \ldots+a_{n} b_{n}}{n}\right) \rightarrow a b$
c) $\left(a_{n}\right)$ is a sequence of positive terms $\Rightarrow \lim _{n \rightarrow \infty} a_{n}^{1 / n}=\lim _{n \rightarrow \infty} \frac{a_{n+1}}{a_{n}}$ if the limit in the right hand side exists whether finite or infinite
d) A sequence $\left(a_{n}\right)$ in R is convergent if and only if it is a Cauchy sequence
7. The Harmonic series $\sum \frac{1}{n^{p}}$ is converges if $\qquad$ and diverges if $\qquad$ -
a) $p<1, p \geq 1$
b) $p>1, p \geq 1$
c) $p>1, p \leq 1$
d) $p<1, p \leq 1$
8. Read the following statement
a) $1+\frac{1}{2}+\frac{1}{3}+\ldots \ldots+\frac{1}{n}+\ldots$. diverges to $\infty$
b) $1+\frac{1}{2}+\left(\frac{1}{2}\right)^{2}+\left(\frac{1}{2}\right)^{3}+$ $\qquad$ . converges to 2

The correct statement is $\qquad$
a) Only (a) is true
c) Both (a) and (b) are true
b) Only (b) is true
d) Neither (a) nor (b) is true
9. The radius of convergence R is
a) $\lim _{n \rightarrow \infty}\left|\frac{a_{n+1}}{a_{n}}\right|$
b) $\lim _{n \rightarrow \infty}\left|\frac{a_{n}}{a_{n+1}}\right|$
c) $\lim _{n \rightarrow 0}\left|\frac{a_{n}}{a_{n+1}}\right|$
d) none
10. If $a_{n}=\frac{2^{n} n!}{n^{n}}$ Then $\lim _{n \rightarrow \infty} \frac{a_{n}}{a_{n+1}}=$ $\qquad$
a) $2 e$
b) $e$
c) $\frac{1}{e}$
d) $\frac{e}{2}$

## $\underline{\text { SECTION - B }}$

## Answer any FIVE Questions

$(5 \times 2=10)$
11. Find the least upper bound and greatest lower bound of the set $A=$ $\left\{\frac{x}{x+1} / x \in N\right\}$.
12. Define the geometric mean and harmonic mean of n real numbers.
13. Define a convergent sequence.
14. Define a cluster point of the sequence.
15. Define a Cauchy sequence.
16. State D'Alembert's ratio test.
17. Show that $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$ converges.

## SECTION - C

## Answer ALL Questions

$(5 \times 5=25)$
18. a) Prove that $a^{7}+b^{7}+c^{7}>a b c\left(a^{4}+b^{4}+c^{4}\right)$.
[OR]
b) State and prove Cauchy-Schwartz inequality.
19. a) If $\left(a_{n}\right) \rightarrow a$ and $\left(a_{n}\right) \rightarrow a$ then prove that $\left(a_{n} b_{n}\right) \rightarrow a b$.

## [OR]

b) Let $a_{n}=1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots+\frac{1}{n!}$ Show that $\lim _{n \rightarrow \infty} a_{n}$ exists and lies between 2 and 3 .
20. a) State and prove Cauchy's general Principle of convergence for sequences.

## [OR]

b) Prove that every sequence an has a monotonic subsequence.
21. a) Discuss the convergence of the series $\sum \frac{1^{2}+2^{2}+3^{2}+\cdots+n^{2}}{n^{4}+1}$.
[OR]
b) State and Prove comparison test.
22. a) Prove that any absolutely convergent series is convergent.

## [OR]

b) Given $\sum \frac{1}{n^{2}}=s$, prove that $1+\frac{1}{3^{2}}+\frac{1}{5^{2}}+\cdots=\frac{3}{4} s$.

## SECTION - D

## Answer any THREE Questions <br> $(3 \times 10=30)$

23. If $x$ is any positive real number and $p, q$ is positive rational then prove that $\frac{x^{p}-1}{p} \geq \frac{x^{q}-1}{q}$ if $p<q$.
24. Discuss the behaviour of the geometric sequence $\left(r_{n}\right)$.
25. Prove that a real number ' $u$ ' is the upper limit of a bounded sequence ( $a_{n}$ ) given $\varepsilon>0(i)$ there exists $n_{0} \in N$ such that $a_{n}<u+\varepsilon$ for all $n \geq n_{0}$.
(ii) there exists infinitely many terms of the sequence $\left(a_{n}\right)$ such that $a_{n}$

$$
>u-\varepsilon
$$

26. State and prove Kummer's test for series.
27. Sate and prove Leibnitz test for series.

$$
5 x^{5} x^{2}
$$

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B.Sc. Mathematics Degree (Semester) Examinations, April 2022 Part - III: Core Course: Fourth Semester: Paper - II

Under CBCS and LOCF - Credit 4
Time: 3 Hours
Max. Marks: 75

## SECTION - A

## Answer ALL Questions

( $\mathbf{1 0} \times \mathbf{1}=\mathbf{1 0})$

1. The horizontal range is maximum when the particle is projected at an angle $\qquad$
a) $\frac{\pi}{2}$
b) $\frac{\pi}{3}$
c) $\frac{\pi}{4}$
d) $2 \pi$
2. The range on the inclined plane is maximum when the angle of projection is $\qquad$
a) $\frac{\pi}{4}$
b) $\frac{\pi}{4}+\frac{\beta}{2}$
c) $\frac{\pi}{4}+\beta$
d) $\frac{\pi}{2}+\frac{\beta}{2}$
3. The velocity of either body resolved in a direction $\qquad$ to the line of impact is not altered by impact
a) Perpendicular
b) parallel
c) any angle
d) any direction
4. A particle falls from a height $h$ upon a fixed horizontal plane. The
$\qquad$ is the velocity of the particle when hitting the plane
a) 2 gh
b) $\sqrt{2 g h}$
c) $\sqrt{u g}$
d) $2 u g$
5. Greatest velocity of S.H.M
a) $\mu$ (amplitude)
b) $\sqrt{\mu}$ (amplitude)
c) $\sqrt{\mu} \mathrm{x}$
d) $\sqrt{\mu}\left(a^{2}-x^{2}\right)$
6. The frequency is the reciprocal of $\qquad$
a) period
b) frequency
c) amplitude
d) velocity
7. A particle describes a path, Attraction by on by an attractive force $F$ towards a fixed point 0 . The path described by the particle is called a
$\qquad$
a) force
b) Attraction
c) central force
d) central orbit.
8. Pedal equation of equiangular spiral
a) $\mathrm{p}=\mathrm{r} \sin \alpha$
b) $\mathrm{p}=\sin \alpha$
c) $\mathrm{p}=\mathrm{r} \cos \alpha$
d) $p=r \tan \alpha$
9. M.I of uniform rectangular parallelepiped of edges $2 \mathrm{a}, 2 \mathrm{~b}$ and 2 c about an axis through the center $O$ parallel to side 2 a is
a) $\frac{M\left(b^{2}+c^{2}\right)}{3}$
b) $\frac{M\left(a^{2}+b^{2}\right)}{3}$
c) $\frac{M\left(a^{2}+c^{2}\right)}{3}$
d) $\frac{M a^{2}}{3}$
10. M.I of a solid right circular cone about its axis is
a) $\frac{M r^{2}}{10}$
(b) $\frac{3 M r^{2}}{5}$
(c) $\frac{3 M r^{2}}{10}$
(d) $\frac{2 M r^{2}}{10}$

## SECTION - B

## Answer any FIVE Questions

$(5 \times 2=10)$
11. Give the formula for greatest height attained by a projectile.
12. Define Principle of conservation of momentum.
13. What are the general solutions of the S.H.M equation?
14. Define a central force.
15. Given the magnitude for radial and transverse component of acceleration.
16. Define the frequency of oscillation.
17. Give the Moments of Inertia of ellipse about the miner axis.

## SECTION - C

## Answer ALL Questions

$(5 \times 5=25)$
18. a) If $h$ and $h^{\prime}$ be the greatest heights in the two paths of a projectile with a given velocity for a given range R , prove that $R=4 \sqrt{h h^{\prime}}$.

## [OR]

b) Find the range on an inclined plane.
19. a) An elastic sphere is projected from a given point O with given velocity V at an inclination $\alpha$ to the horizontal and after hitting a smooth vertical wall at a distance from O returns to O . Prove that $d=\frac{v^{2} \sin 2 \alpha}{g} \frac{e}{1+e}$ where e is the coefficient of restitution.
[OR]
b) A ball of mass 8 gm . moving with a velocity of 10 cm . per sec. impinges directly on another of mass 24 gm ., moving at 2 cm per sec. in the same direction. if $\mathrm{e}=1 / 2$, find the velocities after impact. Also calculate the loss in kinetic energy.
20. a) A particle is moving with S.H.M and while making an oscillation from one extreme position to the other its distances from the centre of oscillation at 3 consecutive seconds are $x_{1}, x_{2}, x_{3}$. Prove that the period of oscillation is $\frac{2 \pi}{\cos ^{-1}\left(\frac{x_{1}+x_{3}}{2 x_{2}}\right)}$.

## [OR]

b) Prove that the length of the equivalent simple pendulum is $\frac{g}{\mu}$
21. a) Derive the formula for perpendicular from the pole on the tangent.
[OR]
b) Derive the pedal equation of a circle with pole at any point.
22. a) Find the moments of inertia of a solid sphere about its diameter.
[OR]
b) Find the moments of inertia of a uniform circular disc about its diameter.

## $\underline{\text { SECTION - D }}$

## Answer any THREE Questions

$$
(3 \times 10=30)
$$

23. Prove that path of a projectile is a parabola.
24. Derive the loss of Kinetic energy due to direct impact of two smooth spheres.
25. Show that the resultant of two simple harmonic motions in the same direction and of equal periodic time, the amplitude of one being twice that of the other and its phase a quarter of a period in advance, is a simple harmonic motion of amplitude $\sqrt{5}$ times that of the first and whose phase is in advance of the first by $\frac{\tan ^{-1} 2}{2 \pi}$ of a period.
26. The velocities of a particle along and perpendicular to a radius vector from a fixed origin are $\lambda r^{2}$ and $\mu \theta^{2}$ where $\mu$ and $\lambda$ are constants. Show that the equation to the path of the particle is $\frac{\lambda}{\theta}+C=\frac{\mu}{2 r^{2}}$, where C is a constant. Show that the acceleration along and perpendicular to the radius vector are $2 \lambda^{2} r^{3}-\frac{\mu^{2} \theta^{4}}{r}$ and $\mu\left(\lambda r \theta^{2}+\frac{2 \mu \theta^{3}}{r}\right)$.
27. Show that the M.I of a triangular lamina of mass $M$ about a side is $\frac{M h^{2}}{6}$ where $h$ is the altitude from the opposite vertex.

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B.Sc. Mathematics Degree (Semester) Examinations, April 2022 Part - III: Core Course: Sixth Semester: Paper - I

LINEAR ALGEBRA
Under CBCS and LOCF - Credit 5
Time: $\mathbf{3}$ Hours

## SECTION - A

## Answer ALL Questions

$(10 \times 1=10)$

1. In $V_{3}(R)$ let $\mathrm{S}=\left\{e_{1}, e_{2}, e_{3}\right\}$ Then $\mathrm{L}(\mathrm{S})=$ $\qquad$
a) S
b) $\{(\mathrm{x}, \mathrm{y}, 0) \mid \mathrm{x}, \mathrm{y} \in R\}$
c) $\{(0, \mathrm{y}, \mathrm{z}) \mid \mathrm{y}, \mathrm{z} \in R\}$
d) $V_{3}(R)$
2. $\operatorname{dim} M_{2}(R)=$ $\qquad$
a) 1
b) 2
c) 3
d) 4
3. Let V be a vector space of polynomial with inner product defined by $<$ $f, g>=\int_{0}^{2} f(t) g(t) d t$ Then the norm of $f$ denoted by $\|f\|$ where $f(t)=$ $t+1$ is given by
a) 9
b) 0
c) $\frac{26}{3}$
d) $\sqrt{\frac{26}{3}}$
4. Given $\left(\begin{array}{ccc}1 & 2 & 3 \\ 0 & -1 & 5\end{array}\right) A\left(\begin{array}{lll}1 & 2 & 3\end{array}\right)=\left(\begin{array}{ccc}1 & 0 & 1 \\ 0 & -1 & 1\end{array}\right)$ The order of the matrix A is
a) $1 \times 3$
b) $3 \times 1$
c) $3 \times 3$
d) $2 \times 2$
5. An example of a symmetric matrix is $\qquad$ .
a) $\left(\begin{array}{lll}1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9\end{array}\right)$
b) $\left(\begin{array}{ccc}1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1\end{array}\right)$
c) $\left(\begin{array}{cc}1 & -1 \\ 1 & 0\end{array}\right)$
d) $\left(\begin{array}{lll}0 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0\end{array}\right)$
6. A square matrix $A$ is said to be involutory if $\qquad$
a) $A^{2}=0$
b) $A^{2}=A$
c) $A^{2}=I$
d) $A^{2}=A^{-1}$
7. If $A=\left(\begin{array}{ll}a & b \\ c & d\end{array}\right)$ and $|A|=a d-b c \neq 0$ then $A^{-1}=$ $\qquad$
a) $\frac{1}{|A|}\left(\begin{array}{cc}d & -b \\ -c & a\end{array}\right)$
b) $\frac{1}{|A|}\left(\begin{array}{cc}-d & b \\ c & -a\end{array}\right)$
c) $\frac{1}{|A|}\left(\begin{array}{ll}-d & -b \\ -c & -a\end{array}\right)$
d) $\frac{1}{|A|}\left(\begin{array}{cc}d & -b \\ -c & a\end{array}\right)$
8. The augmented matrix $(A B)$ of a system of linear equations is reduced to the form $\left(\begin{array}{llll}1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 1\end{array}\right)$ Then...
a) $\operatorname{rank} A=2 ; \operatorname{rank}(A B)=2$
b) $\operatorname{rank} A=2 ; \operatorname{rank}(A B)=3$
c) $\operatorname{rank} A=3 ; \operatorname{rank}(A B)=2$
d) $\operatorname{rank} A=3 ; \operatorname{rank}(A B)=3$
9. The characteristic roots of $\left(\begin{array}{cc}\cos \theta & -\sin \theta \\ -\sin \theta & \cos \theta\end{array}\right)$ are $\qquad$ $-$
a) $-1,-1$
b) 1,1
c) $\cos \theta-\sin \theta ; \cos \theta+\sin \theta$
d) $\sin \theta-\cos \theta ; \sin \theta+\cos \theta$
10. If the eigen values of A are $-1,2,5$ the eigen values of $(5 A)^{-1}$ are $\qquad$
a) $1, \frac{1}{4}, \frac{1}{25}$
b) $-4,14,50$
c) $1,4,25$
d) $-\frac{1}{5}, \frac{1}{10}, \frac{1}{25}$

## SECTION - B

## Answer any FIVE Questions

11. Define subspace
12. Define Linearly Independent
13. Let $V$ be an inner product space and let $S_{1}$ and $S_{2}$ be subsets of $V$. Then prove that $\mathrm{S}_{1} \subseteq \mathrm{~S}_{2} \Rightarrow S_{2}^{\perp} \subseteq S_{1}^{\perp}$
14. Let A be any square matrix. Then prove that $\mathrm{A}-\mathrm{A}^{\mathrm{T}}$ is skew symmetric.
15. Define invertible.
16. Define Eigen vector.
17. Define bilinear form.

## $\underline{\text { SECTION - C }}$

## Answer ALL Questions

$(5 \times 5=25)$
18. a) Prove that the intersection of two subspaces of a vector space is a subspace.

## [OR]

b) Prove that a finite dimensional vector space has a basis consisting of a finite of number of vectors.
19. a) Prove that $\|x+y\| \leq\|x\|+\|y\|$.

## [OR]

b) If S is any subspace of V then prove that $S^{\perp}$ is a subspace of V .
20. a) Show that the matrix $A=\left(\begin{array}{ccc}2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4\end{array}\right)$ satisfies the equation $A(A-$ I) $(A+2 I)$.

## [OR]

b) Compute the inverse of the matrix $\mathrm{A}=\left(\begin{array}{ccc}2 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2\end{array}\right)$
21. a) If $P$ and $A$ are $n \times n$ matrices and $P$ is non-singular matrix then prove that A and $\mathrm{P}^{-1} \mathrm{AP}$ have the same Eigen values.

## [OR]

b) Find the characteristic roots of the equation $A=\left(\begin{array}{cc}\cos \theta & -\sin \theta \\ -\sin \theta & -\cos \theta\end{array}\right)$
22. a) Obtain the matrix representing the Linear Transformation $T: V_{3}(R) \rightarrow$ $\mathrm{V}_{3}(\mathrm{R})$ given by $\mathrm{T}(\mathrm{a}, \mathrm{b}, \mathrm{c})=(3 \mathrm{a}, \mathrm{a}-\mathrm{b}, 2 \mathrm{a}+\mathrm{b}+\mathrm{c})$ with respect to the standard basis ( $\mathrm{e}_{1}, \mathrm{e}_{2}, \mathrm{e}_{3}$ ).
[OR]
b) Find the quadratic form associated with the matrix $\mathrm{A}=\left(\begin{array}{lll}1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 7 & 6\end{array}\right)$

## SECTION - D

## Answer any THREE Questions <br> $(3 \times 10=30)$

23. Let V be a finite dimensional vector space over a field F . Let A and B be the subspaces of V . Then prove that $\operatorname{dim}(A+B)=\operatorname{dim} A+\operatorname{dim} B-\operatorname{dim}(A \cap B)$.
24. Prove that every finite dimensional inner product has an orthonormal basis.
25. Find the rank of the matrix $A=\left(\begin{array}{l}4213 \\ 6347 \\ 2107\end{array}\right)$
26. Find the eigen values and eigen vectors of the matrix $A=\left(\begin{array}{ccc}2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1\end{array}\right)$
27. Reduce the quadratic form $2 \mathrm{x}_{1} \mathrm{x}_{2}-\mathrm{x}_{1} \mathrm{x}_{3}+\mathrm{x}_{1} \mathrm{x}_{4}-\mathrm{x}_{2} \mathrm{x}_{3}+\mathrm{x}_{2} \mathrm{x}_{4}-2 \mathrm{x}_{3} \mathrm{x}_{4}$ to the diagonal form using Lagrange's method.

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B.Sc. Mathematics Degree (Semester) Examinations, April 2022 Part - III: Core Course: Sixth Semester: Paper - II

## SECTION - A

## Answer ALL Questions

$(10 \times 1=10)$

1. Under an inversion a straight line not passing through origin is mapped onto a
a) Straight line passing through the origin
b) Straight line not passing through the origin
c) Circle passing through the origin
d) None
2. A bilinear transformation with only one finite fixed point is called
a) Parabolic
b) Elliptic
c) Hyperbolic
d) None
3. A function $u(x, y)$ is called a harmonic function if it satisfies
a) $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0$
b) $\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}=0$
c) $\frac{\partial^{2} u}{\partial x^{2}}+\frac{\partial^{2} u}{\partial y^{2}}=0$ d) None
4. A function $u(x, y)$ satisfying $\frac{\partial^{2} u}{\partial x^{2}}+\frac{\partial^{2} u}{\partial y^{2}}=0$ is called
a) A harmonic function
b) An entire function
c) An integral function
d) None
5. The integral $\int p d x+q d y$ depends only on the end points of C iff there exist a function $u(x, y)$ such that
a) $\frac{\partial u}{\partial x}=\frac{\partial u}{\partial y}$
b) $\frac{\partial p}{\partial x}=\frac{\partial q}{\partial y}$
c) $p=\frac{\partial u}{\partial x} \& q=\frac{\partial u}{\partial y}$
d) None
6. If $f(z)$ is analytic inside and on a simple closed curve $C$ and $z_{0}$ lies inside C then by Cauchy's integral formula $\frac{1}{2 \pi i} \int_{C} \frac{f(z)}{z-z_{0}} d z$ is
a) Zero
b) $f\left(z_{0}\right)$
c) $f^{1}\left(z_{0}\right)$
d) None
7. By Weierstrass' theorem, the function $f(z)$ comes arbitrarily close to any complex number $\mathbf{c}$ in every neighborhood of $\qquad$ -
a) a removable singularity
b) a pole
c) an essential singularity
d) None
8. If $f(z)$ is analytic in a region $D$ and is not identically zero in $D$ then the set of zeros of $f(z)$ is
a) An empty set
b) a singleton set
c) an isolated set
d) None
9. With usual notations by Rouche's theorem, the two functions which have the same number of zeros inside C are
a) $f(z) \& g(z)$
b) $f(z) \& f(z)+g(z)$
c) $g(z) \& f(z)+g(z)$
d) None
10. The region of convergence of $e^{z}=\sum_{n=0}^{\infty} \frac{z^{n}}{n!}$ is
a) $|z|<1$
b) $|z| \leq 1$
c) $|z|<\infty$
d) None

## $\underline{\text { SECTION - B }}$

## Answer any FIVE Questions

$(5 \times 2=10)$
11. Find the invariant points of the transformation $w=\frac{1+z}{1-z}$.
12. Verify the $C R$ equations for the function $f(z)=e^{x}(\cos y+i \sin y)$
13. Prove that $\left|\int_{c} f(z) d z \leq M l\right|$ where $M=\max \{|f(z)| / z \in C\}$ and $l$ is the length of C .
14. Find the Laurent's series expansion of $f(z)=z^{2} e^{1 / z}$ about $z=0$
15. Calculate the residue of the function $f(z)=\frac{z+1}{z^{2}-2 z}$ at its pole $z=0$
16. Discuss the transformation $\mathrm{w}=\mathrm{z}+\mathrm{b}$
17. Evaluate $\int_{C} \frac{1}{z} \mathrm{dz}$ where C is the circle $|z|=1$

## SECTION - C

## Answer ALL Questions

18. a) Find the bilinear transformation which sends the points $\mathrm{Z}_{1}=0, \mathrm{z}_{2}=-\mathrm{i}$, $\mathrm{Z}_{3}=-1$ into the points $\mathrm{w}_{1}=\mathrm{i}, \mathrm{w}_{2}=1$ and $\mathrm{w}_{3}=0$ respectively.

## [OR]

b) Find the image of the circle $|z-3 i|=3$ under the map $w=\frac{1}{z}$
19. a) State and prove the necessary condition for Cauchy-Riemann equations.

## [OR]

b) If $\mathrm{f}(\mathrm{z})$ is an analytic function of z , show that $\left(\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}\right)|f(z)|^{2}=4$ $\left|f^{\prime}(z)\right|^{2}$
20. a) State and prove Cauchy integral formula.
[OR]
b) Evaluate $\int_{C} \frac{e^{2 z}}{(z+1)^{4}} \mathrm{dz}$ where C is the circle $|z|=2$
21. a) Obtain the Taylor's series to represent $\frac{z^{2}-1}{(z+2)(z+3)}$ in $|z|<2$
[OR]
b) Determine and classify the singular points off $(z)=\frac{z}{e^{z}-1}$
22. a) Using Residue calculus evaluate $\int_{C} \frac{3 \cos z}{2 i-3 z} d z$ where C is the unit circle.

## [OR]

b) Show that $\int_{0}^{\infty} \frac{\cos x}{1+x^{2}} d x=\frac{\pi}{2 e}$.

## SECTION - D

## Answer any THREE Questions: <br> $$
(3 \times 10=30)
$$

23. Prove that a bilinear transformation $\mathrm{w}=\frac{a z+b}{c z+d}$ where $\mathrm{ad}-\mathrm{bc} \neq 0 \mathrm{maps}$ the real axis into itself iff $a, b, c, d$ are real
24. If $\mathrm{u}=\frac{\sin 2 x}{\cos h 2 y+\cos 2 x}$ find the corresponding analytic function $\mathrm{f}(\mathrm{z})=\mathrm{u}+\mathrm{iv}$.
25. State and prove Maximum modulus theorem.
26. Expand $\frac{-1}{(z-1)(z-2)}$ as a power series in $z$ in the regions
i) $|z|<1 \quad$ ii) $1<|z|<2$
27. Evaluate $\int_{0}^{2 \pi} \frac{d \theta}{5+4 \sin \theta}$

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$$

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B.Sc. Mathematics Degree (Semester) Examinations, April 2022 Part - III: Elective Course: Sixth Semester: Paper - I

GRAPH THEORY
Under CBCS and LOCF - Credit 5

## SECTION - A

## Answer ALL Questions

$(10 \times 1=10)$

1. Every cubic graph has $\qquad$ number of points
a) even
b) odd
c) both a and b
d) none
2. The maximum number of lines among all p point graph with no triangle is $\qquad$
a) $\left[\frac{p^{2}}{2}\right]$
b) $\left[\frac{p^{2}}{4}\right]$
c) $\left[\frac{p^{2}}{3}\right]$
d) $\frac{p^{2}}{2}$
3. A walk is $\qquad$ if all the lines are distinct
a) path
b) cycle
c) trail
d) closed walk
4. A graph $G$ with at least two point is bipartite if all its cycles of $\qquad$ lengths.
a) odd
b) even
c) both a and b
d) none of these
5. A closed trail containing all points and lines is called $\qquad$
a) spanning cycle
b) Eulerian trail
c) not Eulerian trail
d) None of these
6. Every tree has a $\qquad$ consisting of either one point or two adjacent points
a) radius
b) eccentricity
c) centre
d) central point
7. Number of perfect matchings is $K_{n, n}$ is $\qquad$ d) $(n-1)$ !
a) $n$
b) $2 n$
c) $n$ !
8. A graph is polyhedral iff it is planer and $\qquad$ -
a) 3- connected
b) 2- connected
c) 1- connected
d) not connected
9. A wheel has chromatic number $\qquad$ if it has odd number of points
a) 2
b) 3
c) 4
d) 5
10. The edge chromatic number, $\aleph^{\prime}\left(\mathrm{K}_{\mathrm{n}}\right)=$ $\qquad$ if $n$ is odd
a) $n$
b) $\mathrm{n}-1$
c) $\mathrm{n}+1$
d) 0

## SECTION - B

## Answer any FIVE Questions

$(5 \times 2=10)$
11. Draw a Peterson graph.
12. Define adjacency matrix and incidence matrix of a graph.
13. Define degree sequence of a graph.
14. Define block of a graph.
15. Define Eulerian graph and Hamiltonian graph.
16. When two graphs are homographic.
17. Define chromatic partitioning.

## SECTION - C

## Answer ALL Questions

$$
(5 \times 5=25)
$$

18. a) Prove that for any graph $G$ with 6 points $G$ or $G$ - contains a triangle. [OR]
b) Prove that every graph is an intersection graph.
19. a) Prove that a closed walk of odd length contains a cycle.
[OR]
b) Prove that a graph $G$ with at least two points is bipartite if all its cycles are of even length.
20. a) Prove that the following statements are equivalent for a connected graph G.
i) $G$ is Eulerian.
ii) Every point of $G$ has even degree.
iii) The set of edges of $G$ can be partitioned into cycles.
[OR]
b) If $G$ is a graph with $\mathrm{p} \geq 3$ vertices and $\delta \geq \mathrm{p} / 2$ then prove that G is Hamiltonian.
21. a) Prove that a matching $M$ in a graph $G$ is a maximum matching if and only if G contains no M -augmenting path.
[OR]
b) If $G$ is connected plane graph having $V, E$ and $F$ as the sets of vertices, edges and faces respectively then prove that $|\mathrm{V}|-|\mathrm{E}|+|\mathrm{F}|=2$.
22. a) Prove that if $G$ is a tree with $n$ points, $n \geq 2$ then $f(G, \lambda)=\lambda(\lambda-1)^{n-1}$.

## [OR]

b) If G is uniquely n -colourable then prove that $\delta(\mathrm{G}) \geq \mathrm{n}-1$.

## SECTION - D

## Answer any THREE Questions

$(3 \times 10=30)$
23. Prove that the maximum number of lines among all p point graphs with no triangles is [ $\left.\mathrm{p}^{2} / 4\right]$.
24. Let G be a connected graph with at least three points. Then prove that the following statements are equivalent.
i) $G$ is a block.
ii) Any two points of G lie on a common cycle.
iii) Any point and any line of G lie on a common cycle.
iv) Any two lines of G lie on a common cycle.
25. Let $G$ be a $(p, q)$ graph. Then prove that the following statements are equivalent.
i) $G$ is a tree.
ii) Every two points of $G$ are joined by a unique path.
iii) G is connected and $\mathrm{p}=\mathrm{q}+1$.
iv) G is acyclic and $\mathrm{p}=\mathrm{q}+1$.
26. State and prove Hall's Marriage theorem.
27. Prove that every planar graph is 5-colourable.

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B.Sc. Mathematics Degree (Semester) Examinations, April 2022 Part - III: Elective Course: Sixth Semester: Paper - II

## SECTION - A

## Answer ALL Questions

$(10 \times 1=10)$

1. If EOQ is calculated, but an order is than placed which is smaller than this, will the total inventory cost
a) Increases
b) decreases
c) either increases or decreases
d) no change
2. Demand is denoted by $\qquad$
a) $C_{0}$
b) $C_{1}$
c) $C_{k}$
d) D
3. The calling population is assumed to be infinite, when
a) arrivals are independent of each other
b) arrivals are dependent upon each other
c) capacity of the system is infinite
d) service rate is faster than arrival rate
4. In Model I, $E(n)=$ $\qquad$
a) $\frac{\lambda}{\mu-\lambda}$
b) $\frac{\mu}{\mu-\lambda}$
c) $-\frac{\lambda}{\mu-\lambda}$
d) $-\frac{\mu}{\mu-\lambda}$
5. The term commonly used for activity slack time is
a) free float
b) independent float
c) total float
d) all the above
6. In a network scheduling, $E_{n}=$
a) $L_{1}$
b) $L_{n}$
c) $L_{2}$
d) $L_{2}$
7. If $A_{i}, B_{i}$ and $C_{i}$ denote the processing times of $i^{\text {th }}$ job on three machine $\mathrm{A}, \mathrm{B}$ and C respectively then a n - job three machine problem can be reduced to n - job two machine problem, provided that
a) $\min A_{i} \geq \max B_{i}$ and/ormin $C_{i} \leq \max B_{i}$
b) $\min A_{i} \geq \max B_{i}$ and/ormin $C_{i} \geq \max B_{i}$
c) $\min A_{i} \leq \max B_{i}$ and/ormin $C_{i} \leq \max B_{i}$
d) $\min A_{i} \leq \max B_{i}$ and/ormin $C_{i} \geq \max B_{i}$
8. The total cost is divided by the number of years is denoted by
a) $e(t)$
b) $S(n)$
c) $T C$
d) $A(n)$
9. When time value of money is considered
a) Costs need to be discounted
b) Timing of incurrence of costs is important
c) The present value factors served as the weights
d) All of the above
10. In a replacement problem, the capital cost is denoted by $\qquad$
a) $C$
b) $S$
c) $f(t)$
d) $A(n)$

## $\underline{\text { SECTION - B }}$

## Answer any FIVE Questions

$(5 \times 2=10)$
11. What is EOQ?
12. Define inventory control.
13. What are the elements of a queueing system?
14. Name the two basic planning and control techniques that utilize a network to complete a pre-determined project or schedule.
15. Define total float of an activity.
16. What is meant by No passing rule in sequencing?
17. Name the classifications of the replacement problems.

## SECTION - C

## Answer ALL Questions

$(5 \times 5=25)$
18. a) Explain the terms- movement inventories, anticipation inventories, decoupling inventories.

## [OR]

b) Neon lights in an industrial park are replaced at the rate of 100 units per day. The physical plant orders the neon lights periodically. It costs ₹ 100 to initiate a purchase order. A neon light kept in storage is estimated to cost about Re. 0.02 per day. The lead time between placing and receiving an order is 12 days. Determine the optimum inventory policy for ordering the neon lights.
19. a) A T.V repairmen finds that the time spent on his jobs has an exponential distribution with mean 30 minutes. If he repairs sets in order in which they came in and if the arrival of sets is approximately Poisson with an average of 10 per 8 -hour day, what is the repairmen's expected idle time each day? How many jobs are ahead of the average set just brought in?

## [OR]

b) Assume that the goods train are coming in a yard at a rate of 30 trains per day and suppose that the inter-arrival times follow an exponential distribution. The service time for each train is assumed to be exponential with an average of 36 minutes. If the yard can admit 9 trains at a time, calculate the probability that the yard is empty and find the average length.
20. a) Draw the network diagram for the following data:

| Activity: | A | B | C | D | E | F | G | H | I | J |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Preceding <br> Activities: | None | A | A | B | A | B,E | C | D,F | G | H,I |

b) A small project consists of seven activities for which the relevant data are given below:

| Activity: | A | B | C | D | E | F | G |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Preceding Activities: | - | - | - | A,B | A,B | C,D,E | C,D,E |
| Activity Duration: | 4 | 7 | 6 | 5 | 7 | 6 | 5 |

i) Draw the network and find the project completion time.
ii) Calculate total float for each of the activities and highlight the critical path.
21. a) A readymade garment company has to process 7 items through two stages of production namely cutting and sewing. The time taken by each of these items at the different stages are given as below:

| Item: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cutting: | 4 | 8 | 3 | 5 | 5 | 12 | 7 |
| Sewing: | 3 | 5 | 6 | 4 | 8 | 5 | 8 |

Find an order in which seven items are to be processed so as to minimize the total processing time.

## [OR]

b) Determine the optimal sequence of jobs that minimizes the total elapsed time based on the following information processing time on machines is given in hours and passing is not allowed:

| Job: | A | B | C | D | E | F | G |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Machine $\mathrm{M}_{1}:$ | 3 | 8 | 7 | 4 | 9 | 8 | 7 |
| Machine $\mathrm{M}_{2}:$ | 4 | 3 | 2 | 5 | 1 | 4 | 3 |
| Machine $\mathrm{M}_{3}:$ | 6 | 7 | 5 | 11 | 5 | 6 | 12 |

22. a) A firm is considering replacement of a machine, whose cost price is $₹ 12,200$ and the scrap value $₹ 200$. The running cost in rupees are found from experience to be as follows:

| Year: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Running <br> Cost: | 200 | 500 | 800 | 1200 | 1800 | 2500 | 3200 | 4000 |

When should the machine be replaced?

## [OR]

b) The cost of a new machine is ₹ 5000 . The maintenance cost of $n^{\text {th }}$ year is given by $C_{n}=500(n-1), n=1,2 \ldots$ Suppose that the discount rate per year is 0.5 . After how many years it will be economical to replace the machine by a new one?

## $\underline{\text { SECTION - D }}$

## Answer any THREE Ouestions <br> $\mathbf{( 3 \times 1 0 = 3 0 )}$

23. A manufacturing company needs 2,500 units of a particular component every year. The company buys it at a rate of ₹ 30 per unit. The order processing cost for this part is estimated at ₹ 15 and the cost of carrying a part in the stock comes to about ₹ 4 per year. The company can manufacture this part internally. In that case, it saves $20 \%$ of the price of the product. However, it estimates a set up cost of ₹ 250 per production run. The annual production rate would be 4800 units. However, the inventory holding costs remain unchanged.
i) Determine the EOQ and the optimal number of orders placed in a year.
ii) Determine the optimum production lot size and the average duration of the production run.
iii) Should the company manufacture the component internally or continue to purchase it from the supplier?
24. A supermarket has a single cashier. During the peak hours, customers arrive at a rate of 20 customers per hour. The average number of customers that can be processed by the cashier is 24 per hour. Calculate:
i) Probability that a cashier is idle.
ii) Average number of customers in the queuing system.
iii) Queue size.
iv) Average time a customer spends in the system.
v) Average time a customer spends in the queue waiting for service.
25. A project consists of eight activities with the following relevant information:

| Activity | Immediate | Estimated duration (days) |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Predecessor | Optimistic | Most likely | Pessimistic |
| A | - | 1 | 1 | 7 |
| B | - | 1 | 4 | 7 |
| C | - | 2 | 2 | 8 |
| D | A | 1 | 1 | 1 |
| E | B | 2 | 5 | 14 |
| F | C | 2 | 5 | 8 |
| G | D,E | 3 | 6 | 15 |
| H | F,G | 1 | 2 | 3 |

i) Draw the PERT network and find out the expected project completion time.
ii) What duration will have $95 \%$ confidence for project completion?
iii) If the average duration for activity $F$ increases to 14 days, what will be its effect on the expected project completion time which will have $95 \%$ confidence?
26. Solve the following sequencing problem when passing out is not allowed.

| Item | Machine (Processing time in hours) |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | A | B | C | D |
| I | 15 | 5 | 4 | 15 |
| II | 12 | 2 | 10 | 12 |
| III | 16 | 3 | 5 | 16 |
| IV | 17 | 3 | 4 | 17 |

27. A manufacturer is offered two machines A and B. A is priced at ₹ 5000 and running costs are estimated at $₹ 800$ for each of the first five years, increasing by ₹ 200 per year in the sixth and subsequent years. Machine B which has the same capacity as A, costs ₹ 2500 but will have running costs of ₹ 1200 per year for six years, increasing by ₹ 200 per year thereafter. If money is worth $10 \%$ per year, which machine should be purchased?

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B.A. \& B.Sc. Degree (Semester) Examinations, April 2022 Part - IV: Generic Elective Course: Second Semester: Paper - I STATISTICS AND OPERATION RESEARCH

Under CBCS and LOCF - Credit 2
Time: 2 Hours
Max. Marks: 75

## SECTION - A

## Answer ALL Questions

$(10 \times 1=10)$

1. The mean of an observation is $\qquad$
a) $\bar{x}=\frac{\sum x}{n}$
b) $\bar{x}=\frac{x+1}{n}$
c) $\bar{x}=\frac{\sum y}{n}$
d) $\bar{x}=\frac{\sum x^{2}}{n}$
2. What is the mode of the sample $5,5,11,9,8,5,8$ ?
a) 5
b) 8
c) 11
d) 9
3. The square root of the variance of a distribution is $\qquad$ $?$
a) Mean
b) Quartile deviation
c) Mode
d) Standard deviation
4. Relation between mean median and mode.
a) 2 Mean + Mode $=3$ Median
b) Mean + Mode $=$ Median
c) Mean +2 Mode $=3$ Median
d) 3 Mean +2 Mode $=$ Median
5. The average of two numbers 50 and 100
a)100
b) 50
c) 75
d) None of these
6. The observation which occurs most frequently in a sample is the $\qquad$ -
a) median
b) mean
c) mode
d) standard deviation
7. Linear Programming Problem (LPP) must have an

## $\underline{\text { SECTION - C }}$

## Answer ALL Questions

18. a) Calculate the Arithmetic mean from the following data:

| Weight in Kg | 50 | 48 | 46 | 44 | 42 | 40 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No of Persons | 12 | 14 | 16 | 13 | 11 | 9 |
| $[O R]$ |  |  |  |  |  |  |

b) Find the median and quartile marks of 10 students in statistics test given by $40,90,61,68,72,43,50,84,75,33$.
19. a) Find standard deviation of marks obtained by 10 students in a test in

Mathematics exam is $80,70,40,50,90,60,100,60,30$, and 80 .
[OR]
b) Find the Standard deviation of the given data:

| Class <br> Interval(x) | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ | $60-70$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency(f) | 1 | 4 | 17 | 45 | 26 | 5 | 2 |

20. a) Use the graphical method to solve the following LPP

Minimize $z=x_{1}+2 x_{2}$
$-x_{1}+3 x_{2} \leq 10$
subject to constraints $\quad x_{1}+x_{2} \leq 6$
$x_{1}-x_{2} \leq 2$.
[OR]
b) Determine the initial basic feasible solution to the following transportation problem using North-West corner method.

|  | D1 | D2 | D3 | D4 | Available |
| :---: | :---: | :---: | :---: | :---: | :---: |
| O1 | 5 | 3 | 6 | 2 | 19 |
| O2 | 4 | 7 | 9 | 1 | 37 |
| O3 | 3 | 4 | 7 | 5 | 34 |
| Requirement | 16 | 18 | 31 | 25 |  |

## $\underline{\text { SECTION - D }}$

## Answer any TWO Questions

$(2 \times 14=28)$
21. Determine the mode of the following distributions.

| Marks | No. of Students | Marks | No. of Students |
| :---: | :---: | :---: | :---: |
| $0-9$ | 6 | $50-59$ | 263 |
| $10-19$ | 29 | $60-69$ | 133 |
| $20-29$ | 87 | $70-79$ | 43 |
| $30-39$ | 181 | $80-89$ | 9 |
| $40-49$ | 247 | $90-99$ | 2 |

22. Compute the arithmetic mean, standard deviation and variance in the following frequency distribution.

| Marks | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 1 | 5 | 11 | 15 | 12 | 7 | 3 | 3 | 0 | 1 |

23. Obtain the initial basic feasible solution to the following transportation problem using North-West corner method.

|  | D | E | F | G | Available |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A | 11 | 13 | 17 | 14 | 250 |
| B | 16 | 18 | 14 | 10 | 300 |
| C | 21 | 24 | 13 | 10 | 400 |
| Requirement | 200 | 225 | 275 | 250 |  |

24. Use the graphical method to solve the following LPP

$$
\begin{aligned}
& \text { Maximize } z=4 x_{1}+3 x_{2} \\
& 2 x_{1}+x_{2} \leq 1000 \\
& x_{1}+x_{2} \leq 800 \\
& x_{1} \leq 400 \text {, and } x_{2} \leq 700 \\
& x_{1}, x_{2} \geq 0 \text {. } \\
& \text { s? }
\end{aligned}
$$

subject to constraints

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B.Sc. Mathematics Degree (Semester) Examinations, April 2022 Part - IV: Skill Based Courses: Sixth Semester: Paper - I

Under CBCS and LOCF - Credit 2
Time: 2 Hours
Max. Marks: 75

## SECTION - A

## Answer ALL Questions

$(10 \times 1=10)$

1. The $\qquad$ class denotes the presence of attributes
a) Frequency
b) Positive
c) Negative
d) Ultimate
2. For any three given attributes total number of positive class frequencies is
a) $2^{3}$
b) $3^{3}$
c) $3^{2}$
d) 7
3. There are no negative class frequencies of order $\qquad$
a) 0
b) 1
c) 2
d) 3
4. For any four given attributes, the total number of positive class frequencies is
a) 15
b) 81
c) 17
d) 16
5. Given $n$ attributes, the total number of class frequencies is
a) $2^{n}$
b) $3^{n}$
c) $2^{n}-1$
d) None
6. Class frequencies of type $(\mathrm{A}),(\mathrm{B}),(\mathrm{ABC}) \ldots$ are known as $\qquad$ class frequencies.
a) Positive
b) Negative
c) Contrary
d) None
7. Two attributes A and B are said to be $\qquad$ if there is same proportion of A as amongst B's
a) Independent
b) Positively associated
c) Negatively associated
d) None
8. Equivalent positive class condition for $(\mathrm{ABC}) \geq 0$.
a) $(\mathrm{ABC}) \leq \mathrm{AB}$
b) $(\mathrm{ABC}) \leq \mathrm{BC}$
c) $(\mathrm{ABC}) \leq \mathrm{AC}$
d) None
9. $\qquad$ is used at determining whether there is a significant difference between class means in view of variability within the separate class.
a) ANOVA
b) null hypothesis
c) alternate hypothesis
d) none
10. The following are ultimate frequencies of two attributes A and $\mathrm{B}, \mathrm{AB}=$ 975, $\alpha B=100, A \beta=25 \& \alpha \beta=950$, then $\beta$ is $\qquad$
a) 975
b) 875
c) 925
d) 950

## SECTION - B

## Answer any FIVE Questions

$(5 \times 2=10)$
11. Given $(A)=(\alpha)=(B)=(\beta)=N / 2$ show that $(i)(A B)=(\alpha \beta) \quad$ (ii) $(A \beta)=(\alpha B)$
12. Define consistency.
13. Find whether the following data are consistent for $\mathrm{A}=300, \mathrm{~B}=400$, $A B=50 \& N=600$.
14. Check whether the attributes A and B are independent for $\mathrm{A}=30, \mathrm{~B}=$ $60, \mathrm{AB}=12 \& \mathrm{~N}=15$
15. Prove that $(A B)=A B C+(A B x)$
16. Prove that $(\mathrm{AB})=N-\alpha-\beta+(\alpha \beta)$.
17. Define inconsistent.

## $\underline{\text { SECTION - C }}$

## Answer ALL Questions

18. a) In a class test in which 135 candidates were examined for proficiency in English and Maths. It was discovered that 75 students failed in English, 90 failed in Maths and 50 failed in both. Find how many candidates: i) have passed in Maths ii) have passed in English \& failed in Maths \& iii) have passed in both.

## [OR]

b) Given $N=1200, A B C=600,(\alpha \beta x)=50, \gamma=270,(A \beta)=36,(B \gamma)=$ 204, $A-\alpha=192 \& B-\beta=620$. Find the remaining ultimate class frequencies.
19. a) Find the greatest and least value of $(A B C)$ if $(A)=50 ;(B)=62 ;(C)=$ $80 ;(\mathrm{AB})=35 ;(\mathrm{AC})=45$ and $(\mathrm{BC})=42$.
[OR]
b) Of 2000 people consulted 1854 speak tamil, 1507 speak hindi, 572 speak English, 676 speak tamil and hindi, 286 speak tamil and English, 270 speak hindi and English, 114 speak tamil, hindi \& English. Show that the information as it stands is incorrect.
20. a) Show whether A and B are independent (or) positively associated (or) negatively associated in the following cases. i) $A=470, A B=300$, $\alpha=530, \alpha \mathrm{~B}=150 \&$ ii) $\mathrm{AB}=66, \mathrm{~A} \beta=88, \alpha \mathrm{~B}=102, \alpha \beta=136$.

## [OR]

b) Calculate the coefficient of association between intelligence of father and son from the following data.

Intelligent fathers with intelligent sons 200
Intelligent fathers with dull sons 50
Dull fathers with intelligent sons 110
Dull fathers with dull sons 600. Comment on the result.

## $\underline{\text { SECTION－D }}$

## Answer any TWO Questions

（ $2 \times 14=28$ ）
21．The following is the statistics showing the lives in hours of four batches of electric bulbs sold in different shops．Perform an analysis of variance and state your conclusion．

| Batches | S1 | S2 | S3 | S4 | S5 | S6 | S7 | S8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 1600 | 1610 | 1650 | 1680 | 1700 | 1720 | 1800 | - |
| B | 1580 | 1640 | 1700 | 1750 | 1640 | - | - | - |
| C | 1460 | 1550 | 1600 | 1620 | 1640 | 1660 | 1740 | 1820 |
| D | 1510 | 1520 | 1530 | 1570 | 1600 | 1680 | - | - |

22．Given the following positive class frequencies $\mathrm{N}=20, \mathrm{~A}=9, \mathrm{~B}=12$ ， $\mathrm{C}=8, \mathrm{AB}=6, \mathrm{BC}=4, \mathrm{CA}=4 \& \mathrm{ABC}=3$ ，Find the remaining Class frequencies．

23．Show that for n attributes $\mathrm{A}_{1}, \mathrm{~A}_{2}, \ldots . \mathrm{A}_{\mathrm{n}} .\left(\mathrm{A}_{1}, \mathrm{~A}_{2} \ldots . . \mathrm{A}_{\mathrm{n}}\right) \geq\left(\mathrm{A}_{1}\right)+\left(\mathrm{A}_{2}\right)+$ $\ldots .+\left(A_{n}\right)-(n-1) N$ ，where $N$ is the total number of observations．

24．From the following data compare the association between marks in physics and chemistry in M．K．U and M．S．U．

| University | M．K．U | M．S．U |
| :--- | :---: | :---: |
| Total Number of candidates | 1600 | 200 |
| Pass in Physics | 320 | 80 |
| Pass in Chemistry | 90 | 40 |
| Pass in Physics \＆Chemistry | 30 | 20 |

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## SECTION - A

## Answer ALL Questions

$(10 \times 1=10)$

1. Let $S=\{1,2,3,4\}$. define a relation $\rho$ on $S$ as a $\rho b \Leftrightarrow a<b$. Then $\rho$ is $\qquad$ -.
a) $\{(1,2)(1,3)(1,4)\}$
b) $\{(1,2),(1,3),(1,4),(2,3),(2,4)\}$
c) $\{(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)\}$
d) $\{(1,1),(2,2),(3,3),(4,4)\}$
2. Let $S$ be the set of all lines in the Euclidean plane $R \times R$. Define a $\rho b \Leftrightarrow$ a is parallel to $b$. Then $\rho$ is $\qquad$
a) not reflexive
b) not symmetric
c) not transitive
d) equivalence relation
3. A lattice is a poset in which any two elements have a $\qquad$ -
a) g.l.b
b) l.u. $b$.
c) g.l.b and l.u.b.
d) None of these
4. The relation $\rho$ is defined on $Z$ as $x \rho y \Leftrightarrow x-y$ is a multiple of 5. Then the equivalence class [2] is
a) $\{5 \mathrm{k}+2 / \mathrm{k} \in Z\}$
b) $\{2 \mathrm{k}+5 / \mathrm{k} \in Z\}$
c) $\{2 \mathrm{k}+5 / \mathrm{k} \in N\}$
d) $\{5 \mathrm{k}+2 / \mathrm{k} \in N\}$
5. The least element of all finite subset of any infinite set is $\qquad$
a)1
b) 2
c) $\phi$
d) None of these
6. The lattice of normal subgroup of any group is a $\qquad$
a) Distributive
b) Unit
c) Complemented
d) Modular
7. The value of $a \wedge a^{\prime}=$ $\qquad$

## SECTION - C

a) 1
b) 0
c) infinite
d) $a$
8. In any Boolean algebra, each of the identifies $a \wedge x=a$ and $a \vee x=x$ for all x implies $\qquad$
a) $a=b$
b) $a=a$ '
c) $a=0$
d) None of these
9. In a distributive lattice the compliment of any element a, if it exists, is
$\qquad$
a) Modular
b) Unique
c) Complemented
d) Distributive
10. Let L be a lattice. Let $a, b \in L$. Then idempotent is $\qquad$ $=\mathrm{a}$
a) $a<b$
b) $a>b$
c) $a \vee a$
d) $a \wedge a$

## SECTION - B

## Answer any FIVE Questions

$$
(5 \times 2=10)
$$

11. Define Reflexive, Symmetric and Transitive.
12. If $\mathrm{P}\{1,2,3,4\}$ with the usual less than or equal, then draw a partial ordering set diagram.
13. Write the formula for distributive lattices.
14. Explain modular lattice.
15. Define Boolean algebra.
16. Describe complemented lattice.
17. Elucidate a Boolean algebra B, the complement of any element is not itself.

## Answer ALL Questions

$(3 \times 9=27)$
18. a) i) If $S=Z, a \rho b$ means $a \equiv b(\bmod m)$ then verify equivalence relations.
ii) Validate equivalence relations if $S=R, a \rho b$ means $a=b$.

## [OR]

b) i) Demonstrate that the union of two equivalence relations need not to be an equivalence relation.
ii) If $\rho$ and $\sigma$ are equivalence relations defined on a set $S$, demonstrate that $\rho \cap \sigma$ is an equivalence relation.
19. a) Let L be any non-empty set with two binary operations $\wedge a n d \vee$ defined on it and satisfying $L_{1}, L_{2}, L_{3}, L_{4}, L_{1}^{\prime}, L_{2}^{\prime}, L_{3}^{\prime}, L_{4}^{\prime}$. Then L is lattice relative to a suitable definition of $\leq, \wedge$ and $\vee$ are the g.l.b and l.u.b in this lattice.

## [OR]

b) Demonstrate that the lattice of normal subgroups of any group is a modular lattice.
20. a) Let $L$ be a Boolean algebra. Then prove that

$$
\text { i) }(a \vee b)^{\prime}=a^{\prime} \wedge b^{\prime} \text { and }(a \wedge b)^{\prime}=a^{\prime} \vee b^{\prime} \quad \text { ii) }\left(a^{\prime}\right)^{\prime}=a
$$

[OR]
b) In a Boolean algebra if $a \vee x=b \vee x$ and $a \vee x^{\prime}=b \vee x^{\prime}$ then $a=b$.

In addition, show that $\left[a \vee\left(a^{\prime} \wedge b\right)\right] \wedge[b \vee(b \wedge c)]=b$.

## $\underline{\text { SECTION - D }}$

## Answer any TWO Questions

21. If $\rho$ bean equivalence relation defined on a set S . Then
i) $a \rho b \Leftrightarrow[a]=[b]$.
ii) Any two distinct equivalence classes are disjoint.
iii) $S$ is the union of all equivalence classes.
22. Describe the covers of following sets and draw a poset diagram, if the set of all subgroup of
i) $V_{4}=\{e, a, b, c\}$ given by $\{\mathrm{e}\},\{\mathrm{e}, \mathrm{a}\},\{\mathrm{e}, \mathrm{b}\},\{\mathrm{e}, \mathrm{c}\}$ and $\mathrm{V}_{4}$.
ii) $\{1,2,3\}$.
23. Let L be a lattice. Let $a, b, c \in L$. Then we have
i) $L_{1}: a \vee a=a$ and $L_{1}{ }^{\prime}: a \wedge a=a$
ii) $L_{2}: a \vee b=b \vee a$ and $L_{2} ': a \wedge b=b \wedge a$
iii) $L_{3}: a \wedge(a \vee b)=a$ and $L_{3}{ }^{\prime}: a \vee(a \wedge b)=a$
24. i) Show that in any distributive lattice
$(a \vee b) \wedge(b \vee c) \wedge(c \vee a)=(a \wedge b) \vee(b \wedge c) \vee(c \wedge a)$.
ii) Explain, Boolean algebra cannot have exactly three elements.
Sisisy

# VIVEKANANDA COLLEGE, TIRUVEDAKAM WEST College with Potential for Excellence 

Residential \& Autonomous - A Gurukula Institute of Life-Training Re-accredited (3rd Cycle) with 'A' Grade (CGPA 3.59 out of 4.00 ) by NAAC [Affiliated to Madurai Kamaraj University]
B.A., B.Sc., B.Com. \& B.Com.(CA) Degree (Semester) Examinations, April 2022 COMPETITIVE MATHEMATICS
Time: $\mathbf{2}$ Hours
SECTION - A

## Answer ALL Questions

1. Two numbers are in the ratio of $15: 11$. If their H.C.F is 13 , find the numbers?
2. Find the average of all prime numbers between 30 and 50 ?
3. If $a: b=5: 9$ and $b: c=4: 7$, find $a: b: c$.
4. If $\sqrt{3^{n}}=729$, then the value of $n$ is $\qquad$ .
5. Find $x$ if $\frac{x}{\sqrt{128}}=\frac{\sqrt{168}}{x}$.
6. Divide Rs. 1162 among A, B, C in the ratio 35:28:20.
7. Express $6 \frac{3}{4}$ as rate percent.
8. A man buys an article for Rs. 27.50 and sells it for Rs. 28.60. Find his gain.
9. $\sqrt{248+\sqrt{51 \sqrt{169}}}=$ ?
10.3.5 can be expressed in terms of percentage as $\qquad$ .

## $\underline{\text { SECTION - B }}$

## Answer ALL Questions

11.a) Find the followings:
i) $0.4 \times 0.04 \times 0.04 \times 40=$ ?
ii) $6202.5+620.25+62.025+6.2025+0.62025=$ ?
[OR]
b) There are two sections A and B of a class, consisting of 36 and 44 students respectively. If the average weight of section $A$ is 40 kg and that of section $B$ is 35 kg , find the average weight of the whole class.
12.a) i) One year ago, the ratio of Gaurav's and Sachin's age was $6: 7$ respectively. Four years hence, this ratio would become 7:8. How old is Sachin?
ii) $28 \%$ of $450+45 \%$ of $280=$ ?

## [OR]

b) i) The product of ages of Ankit and Nikita is 240. If twice the age of Nikita is more than Ankit's age by 4 years, what is Nikita's age?
ii) 2 is what percent of 50 ?
13.a) i) If a radio is purchased for Rs. 490 and sold for Rs. 465.50 , find the loss percent.
ii) Find C.P, when S.P $=$ Rs. 40.60 and Gain $=16 \%$.
b) i) Find S.P, when C.P $=$ Rs. 56.25 , Loss $=20 \%$.
ii) if $x: y=3: 4$, find $(4 x+5 y):(5 x-2 y)$.
14.a) A, B, and C enter into partnership. A invests 3 times as much as B invests and B invests two third of what C invests. At the end of the year, the profit earned is Rs. 6600 . What is the share of B ?

## [OR]

b) A bag contains 50 p, 25 p and 10 p coins in the ratio 5:9:4, amounting to Rs. 206. Find the number of coins of each type.

## SECTION - C

## Answer any TWO Questions

$$
(2 \times 10=20)
$$

15.i) Find the L.C.M of 72, 108 and 2100.
ii) Find the HCF of 513, 1134 and 1215.
16.i) Find the square root of 1471369 .
ii) simplify: $\sqrt{\left[(12.1)^{2}-(8.1)^{2}\right] \div\left[(0.25)^{2}+(0.25)(19.95)\right]}$.
iii) Find the average of first 40 natural numbers.
17.i) A, B and C started a business by investing Rs. 120000 , Rs. 135000 and Rs. 150000 respectively. Find the share of each, out of an annual profit of Rs.56700.
ii) The salaries of A, B, C are in the ratio 2:3:5. If the increments of $15 \%, 10 \%$ and $20 \%$ are allowed respectively in their salaries, then what will be the new ratio of their salaries?


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B.A., B.Sc., B.Com. \& B.Com.(CA) Degree (Semester) Examinations, April 2022 MATHEMATICS FOR HIGHER STUDIES
Time: $\mathbf{2}$ Hours
CERTIFICATE COURSES
Max. Marks: 50
SECTION - A

## Answer ALL Questions

1. Find the unit vector perpendicular to the plane of $\vec{u} \& \vec{v}$ ?
2. If a vector field $\vec{F}$ is conservative then the value of $\operatorname{Curl} \vec{F}$ ?
3. Define Leibnitz's rule.
4. In a given curve with equation $x=f(y)$, find the length of the curve between the points $\mathrm{y}=\mathrm{a}$ and $\mathrm{y}=\mathrm{b}$ ?
5. Define separable differential equation.
6. What is the condition for a differential equation $M(x, y) d x+N(x, y) d y=0$ ?
7. Define Rolle's theorem.
8. Define Uniform Continuous function.
9. Define Linear Transformation on a Vector Space.
10. Define Transpose of a Matrix.

## $\underline{\text { SECTION - B }}$

## Answer ALL Questions

11.a) Consider the vector field $\vec{F}=(a x+y+a) \vec{i}+\vec{j}-(x+y) \vec{k}$, where $\mathbf{a}$ is a constant. If
$\vec{F}$.Curl $\vec{F}=0$, then find the value of $\mathbf{a}$ ?
[OR]
b) Let $\vec{F}=a y \vec{i}+z \vec{j}+x \vec{k}$ and C be the positively oriented closed curve given by $x^{2}+y^{2}=1, z=0$.

If $\oint_{C} \vec{F} . d \vec{r}=\pi$, then find the value of $\mathbf{a}$ ?
12.a) Find the value of $\lim _{x \rightarrow 0} \frac{1}{\sin ^{2} x} \int_{x / 2}^{x} \sin ^{-1} t d t$ ?
[OR]
b) Let $f(x)=\int_{\sin x}^{\cos x} e^{-t^{2}} d t$. Then find the value of $f^{\prime}\left(\frac{\pi}{4}\right)$ ?
13. a) Find the particular integral of the differential equation $\frac{d^{2} y}{d x^{2}}-2 \frac{d y}{d x}=e^{2 x} \sin x$.
[OR]
b) Find the solution $y(x)$ of the differential equation $\frac{d^{2} y}{d x^{2}}+4 \frac{d y}{d x}+4 y=0$ satisfying the conditions $y(0)=4, \frac{d y(0)}{d x}=8$.

14．a）Find the value of $\lim _{n \rightarrow \infty} \frac{2^{n+1}+3^{n+1}}{2^{n}+3^{n}}$ ．

## ［OR］

b）Find the number of 11 subgroups of the group $\left(Z_{60},+\right)$ of integers modulo 60 ？

## SECTION－C

## Answer any TWO Questions

$(2 \times 10=20)$
15．Find the surface area of the portion of the plane $y+2 z=2$ within the cylinder $x^{2}+y^{2}=3^{2}$ ？
16．If $y(x)=\lambda e^{2 x}+e^{\beta x}, \beta \neq 2$ ，is solution of the differential equation $\frac{d^{2} y}{d x^{2}}+\frac{d y}{d x}-6 y=0$ satisfying $\frac{d y(0)}{d x}=5$, find the value of $y(0) ?$

17．Let $x_{n}=\left(1-\frac{1}{3}\right)^{2}\left(1-\frac{1}{6}\right)^{2}\left(1-\frac{1}{10}\right)^{2} \ldots\left(1-\frac{1}{\frac{n(n+1)}{2}}\right)^{2}, n \geq 2$ ．Find the value of $\lim _{n \rightarrow \infty} x_{n}$ ．々々民々々

