VIVEKANANDA COLLEGE, TIRUVEDAKAM WEST
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[Affiliated to Madurai Kamaraj University]

B.Sc. (Phy. / Chem.) Degree (Semester) Examinations, April 2017
Part – III : Allied Subject : Fourth Semester : Paper – II

MATHEMATICS – II
Under CBCS – Credit 5

Time: **3** Hours Max. Marks: **75**

SECTION – A

Answer ALL Questions : **(10 × 1 = 10)**

1. The _____ of a differential equation is the order of the highest derivative occurring in it.
   a) order b) degree c) roots d) solution
2. A differential equation is said to be _____ when the dependent variable and its derivatives occur only in the first degree.
3. The solution of the equation $(D^2-5D+4)y=0$ is $y=$ _____.
   a) $C_1e^x+C_2e^{4x}$ b) $4e^x+e^{4x}$ c) e^x+e^{4x} d) $C_1e^{-x}C_2e^{-4x}$
4. $\frac{1}{f(D)}e^{ax}=$ _____.
5. The formula for $L(\cos at)=$ _____.
   a) $\frac{a}{s^2+a^2}$ b) $\frac{s}{s^2+a^2}$ c) $\frac{-a}{s^2+a^2}$ d) $\frac{-s}{s^2+a^2}$
6. $L^{-1}\left[\frac{1}{(s+a)^2}\right]=$ _____.
7. By eliminating a *and* b from $z=(x+a)(y+b)$ *then* $z=$ _____.
   a) ab b) $-ab$ c) $-pq$ d) pq
8. A solution containing as many arbitrary constants as there are independent variables is called a _____.
9. $\int_{-a}^{a} f(x)dx=$ _____.
   a) ∞ b) 1 c) 0 d) 2
10. *If* $f(x)=-f(x)$, *then* $f(x)$ is said to be an _____ function.

SECTION – B

Answer ALL Questions : **(5 × 7 = 35)**

11. a) Solve: $\frac{dy}{dx}-x\tan(y-x)=1.$ **(OR)**
    b) Solve: $xdy-ydx=\sqrt{x^2+y^2}\,dx.$
12. a) Solve: $(D^2+5D+6)y=e^x.$ **(OR)**
    b) Solve: $(D^2+16)y=e^{-3x}+\cos 4x.$
13. a) Find $L(\sin^2 2t).$ **(OR)**
    b) Find $L^{-1}\left[\frac{s}{(s+3)^2+4}\right].$
14. a) Solve: $x\frac{\partial z}{\partial x}=2x+y+3z.$ **(OR)**
    b) Solve: $p^2+q^2=npq.$
15. a) Determine the values of a_0 and a_n for the fourier series of the function $f(x)=\frac{1}{2}(\pi-x)$ with period 2π in the interval 0 *to* 2π.

    (OR)

    b) Express $f(x)=x, -\pi<x<\pi$ as a fourier series with period 0 *to* 2π.

SECTION – C

Answer any THREE Questions : **(3 × 10 = 30)**

16. Solve: $\frac{dy}{dx}=\frac{x+2y-3}{2x+y-3}.$
17. Solve: $(D^2-8D+9)y=8\sin 5x.$
18. Find the inverse transform for i) $\frac{s}{(s-1)^3}$ ii) $\frac{1+2s}{(s+2)^2(s-1)^2}$
19. Solve: i) $q=xp+p^2$ ii) $p=y^2q^2$
20. Find a sine series for $f(x)=c$ in the range 0 *to* π.

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B.Sc. Mathematics Degree (Semester) Examinations, April 2017
Part – III : Allied Subject : Fourth Semester : Paper – I

PROGRAMMING IN C++
Under CBCS – Credit 3

Time: **3** Hours Max. Marks: **75**

SECTION – A

Answer ALL Questions : **(10 × 1 = 10)**

1. The wrapping up of data and functions into a single unit is known as _____.
   a) Encapsulation b) polymorphism
   c) dynamic binding d) message communication
2. _____ is the process by which objects of one class acquire the properties of objects of another class.
3. If we can use the function name to create functions that perform a variety of different tasks then this function is known as _____.
   a) main function b) function polymorphism
   c) function overloading d) friend function
4. The explicit use of a _____ statement will indicate that the program was successfully executed.
5. The variables declared inside the class are known as _____.
   a) member functions b) members
   c) data members d) data hiding
6. The function _____ displays the code number of each object.
7. _____ member function enables an object to initialize itself when it is created.
   a) destructor b) constructor c) put data() d) get data()
8. A friend function will have _____ argument for unary operators.
9. The mechanism of deriving a new class from an old one is called _____.
   a) inheritance b) base class c) abstract class d) derived class
10. Always, the default visibility – mode is _____.

SECTION – B

Answer ALL Questions : **(5 × 7 = 35)**

11. a) Explain the applications of OPP in C++. **(OR)**
    b) Explain the structure of C++ program.
12. a) Explain about Return by Reference function in C++. **(OR)**
    b) Explain about Default arguments in C++.
13. a) How to define member functions? **(OR)**
    b) Explain about static member functions.
14. a) Explain about Constructors with an example. **(OR)**
    b) Explain about Constructors with default arguments.
15. a) Explain about Multilevel Inheritance. **(OR)**
    b) Explain about Multiple Inheritance.

SECTION – C

Answer any THREE Questions : **(3 × 10 = 30)**

16. Discuss about OOP's concept in detail.
17. Explain about Call by Reference function in detail.
18. Explain about Friend functions in C++.
19. Explain about Destructors with an example.
20. Explain about Hierarchical Inheritance and Hybrid Inheritance.

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B.Sc. Mathematics Degree (Semester) Examinations, April 2017
Part – III : Core Subject : Second Semester : Paper – I

INTEGRAL CALCULUS
Under CBCS – Credit 4

Time: **3** Hours Max. Marks: **75**

SECTION – A

Answer ALL Questions : **(10 × 1 = 10)**

1. $\int \tan\theta \, d\theta =$

   a) $\log(\sec\theta)$ b) $\log(\cos\theta)$ c) $\log(\sin\theta)$ d) $\log(\cos ec\theta)$

2. The ultimate integral value of $\int e^{ax} \, dx$ is

   a) $\frac{e^{ax}}{-a}$ b) $\frac{e^{ax}}{a}$ c) ae^{ax} d) e^{ax}

3. $\overline{|(n+a)|} =$

   a) $\overline{|(n)|}$ b) $n\overline{|(n)|}$ *if* $n<0$ c) $n\overline{|(n)|}$ *if* $n>0$ d) None

4. $\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} \end{vmatrix} =$ _____.

   a) $\frac{\partial(u,v)}{\partial(x,y)}$ b) $\frac{\partial(x,y)}{\partial(u,v)}$ c) $\frac{\partial(u,u)}{\partial(x,y)}$ d) $\frac{\partial(v,v)}{\partial(x,y)}$

5. $\int_0^{\pi} \cos mx \, dx =$

   a) 0 b) π c) $\pi/2$ d) None

6. $\int \sin(ax+b)\,dx =$ _____.

7. $\int_a^c f(x)\,dx + \int_c^b f(x)\,dx =$ _____.

8. $\iint f(x,y)\,dx\,dy = \iint F(u,v)$ _____.

9. The value of $\frac{\partial(u,v)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(u,v)} =$ _____.

10. The Fourier coefficient a_0 in the interval $-\pi$ *to* π is _____.

SECTION – B

Answer ALL Questions : **(5 × 7 = 35)**

11. a) Evaluate $\int \sin^4 x\,dx$.

(OR)

b) Evaluate $\int \frac{dx}{\sqrt{(3x^2+x-2)}}$.

12. a) Find the reduction formula for $\int \sin^n x\,dx$.

(OR)

b) Evaluate $\int x \sin 2x\,dx$.

13. a) Evaluate $\iint xy\,dxdy$ taken over the positive quadrant of the circle $x^2+y^2=a^2$.

(OR)

b) Prove that $\beta(m,n) = \int_0^{\frac{\pi}{2}} \sin^{2m-1} x \cos^{2n-1} x\,dx$.

14. a) Find the area of the cardioids $r = a(1+\cos\theta)$.

(OR)

b) Prove that $\frac{\partial(x,y,z)}{\partial(r,\phi,\theta)} = -r^2 \sin\theta$ where x, y, z are spherical polar coordinates.

15. a) Find the Fourier series for the function $f(x) = x,\ x \in (-\pi, \pi)$.

(OR)

b) Find the sine series for $f(x) = c$ in the range $(0, \pi)$.

SECTION – C

Answer any THREE Questions : **(3 × 10 = 30)**

16. Evaluate $\int \frac{2x+3}{x^2+x+1}\,dx$.

17. Evaluate $\int_0^{\pi/2} \frac{(\sin x)^{\frac{3}{2}}}{(\sin x)^{\frac{3}{2}} + (\cos x)^{\frac{3}{2}}}\,dx$.

18. Obtain the relationship between beta and gamma function.

19. Evaluate $\iiint xyz\,dx\,dy\,dz$ taken through the positive octant of the sphere $x^2+y^2+z^2=a^2$.

20. Find the Fourier series for $f(x) = \frac{1}{2}(\pi - x)$ in the period 0 *to* 2π.

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B.Sc. Mathematics Degree (Semester) Examinations, April 2017
Part – III : Core Subject : Second Semester : Paper – II

ANALYTICAL GEOMETRY 3D AND VECTOR CALCULUS
Under CBCS – Credit 4

Time: **3** Hours Max. Marks: **75**

SECTION – A

Answer ALL Questions : **(10 × 1 = 10)**

1. *If* α, β, γ be the angles which any line makes with the positive direction of the axes, then $\cos\alpha, \cos\beta, \cos\gamma$ are called the direction _____ of the given line.
   a) cosines b) ratios c) points d) none
2. The equation which is satisfied by the coordinates of any point in the plane is called the equation of the _____.
3. A straight line may be determined as the intersection of two _____.
   a) lines b) planes c) circles d) spheres
4. Two straight lines in space which are not coplanar are called _____.
5. A _____ is the locus of a point which moves in such a way that its distance from a fixed point is always constant.
   a) lines b) planes c) circles d) sphere
6. The plane section of a sphere is a _____.
7. The dot product is _____.
   a) Additive b) commutative c) multiplicative d) associative
8. The gradient of a scalar function φ is a _____ valued function.
9. Green's theorem in space is same as _____ theorem.
   a) Stoke's b) Green's c) Gauss divergence d) Euler's
10. By the definition of Stoke's theorem, $\int_C \vec{f}.d\vec{r} =$ _____.

SECTION – B

Answer ALL Questions : **(5 × 7 = 35)**

11. a) Find the direction cosines of the line joining the points $(3,-5,4)$ *and* $(1,-8,-2)$.

(OR)

b) Find the angle between the planes $2x-y+z=6,\ x+y+2z=3.$

12. a) Find the perpendicular distance from $P(3,9,-1)$ to the line $\frac{x+8}{-8}=\frac{y-31}{1}=\frac{z-31}{5}$.

(OR)

b) Prove that the lines $\frac{x+1}{-3}=\frac{y+10}{8}=\frac{z-1}{2};\ \frac{x+3}{-4}=\frac{y+1}{7}=\frac{z-4}{1}$ are coplanar.

13. a) Find the co-ordinates of the centre and radius of the sphere $2x^2+2y^2+2z^2-2x+4y+2z-15=0.$

(OR)

b) Find the equation of a sphere which touches the sphere $x^2+y^2+z^2-6x+2z+1=0$ at the point $(2,-2,1)$ and passes through the origin.

14. a) *If* $\vec{r}=\vec{a}\cos\omega t+\vec{b}\sin\omega t$ where $\vec{a},\vec{b}$ are constant vectors and ω is a constant then prove that $\vec{r}\times\frac{d\vec{r}}{dt}=\omega(\vec{a}\times\vec{b})$ *and* $\frac{d^2\vec{r}}{dt^2}+\omega^2\vec{r}=0.$

(OR)

b) *If* $\vec{r}$ is the position vector of any point $p(x,y,z)$ then, prove that grad $r^n=n\,r^{n-2}\,\vec{r}.$

15. a) Find the work done by the force $\vec{F}=3xy\vec{i}-5z\vec{j}+10x\vec{k}$ along the curve $C,\ x=t^2+1,\ y=2t^2,\ z=t^3$ from $t=1$ *to* $t=2.$

(OR)

b) Verify green's theorem for the function $\vec{f}=(x^2+y^2)\vec{i}-2xy\vec{j}$ *and* C is the rectangle in the $x-y$ plane bounded by $y=0,\ y=b,$ $x=0$ *and* $x=a.$

SECTION – C

Answer any THREE Questions : **(3 × 10 = 30)**

16. Find the equation of the plane which passes through the point $(-1,3,2)$ *and* perpendicular to the two planes $x+2y+2z=5,$ $3x+3y+2z=8.$

17. Find the shortest distance between the lines $\frac{x-3}{-1}=\frac{y-4}{2}=\frac{z+2}{1};\ \frac{x-1}{1}=\frac{y+7}{3}=\frac{z+2}{2}.$

18. Find the equation to the sphere through the four points $(2,3,1),\ (5,-1,2),\ (4,3,-1)$ *and* $(2,5,3).$

19. Show that *div* $\left(\frac{\vec{r}}{r}\right)=\frac{2}{r}.$

20. Evaluate $\iint_S \vec{f}.\vec{n}\ dS$ where $\vec{f}=(x^2+y^2)\vec{i}-2x\vec{j}+2yz\vec{k}$ *and* S is the surface of the plane $2x+y+2z=6$ is the first octant.

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B.Sc. Mathematics Degree (Semester) Examinations, April 2017
Part – III : Core Subject : Fourth Semester : Paper – I

SEQUENCE AND SERIES
Under CBCS – Credit 5

Time: **3** Hours Max. Marks: **75**

SECTION – A

Answer ALL Questions : **(10 × 1 = 10)**

1. A subset A of R is said to be _____ if there exists an element $a \varepsilon R$ such that $a \leq \alpha$ for all $\alpha \varepsilon A$.
   a) bounded below b) bounded above
   c) unbounded d) none of these
2. $f : A \to R$ is said to be a _____ if its range is a bounded subset of R.
   a) bounded function b) unbounded function
   c) onto function d) none of these
3. A sequence (a_n) is said to be _____if there exists a real number k such that $a_n \geq k$ for all n.
   a) bounded above b) bounded below
   c) unbounded d) none of these
4. A sequence (a_n) is said to be _____ *if* $a_n \leq a_{n+1}$ for all n.
   a) monotonic increasing b) monotonic decreasing
   c) oscillating d) none of these
5. *Let* (a_n) be a sequence. *Let* (n_k) be a strictly increasing sequence of natural numbers. Then (a_{n_k}) is called a _____of (a_n).
   a) sequence b) subsequence c) subset d) none of these
6. Every bounded sequence has atleast _____ limit point.
   a) one b) two c) three d) four

7. The series $\sum a_n$ is said to _____according as the sequence of partial sums (S_n) converges.

a) diverge b) oscillate c) converge d) none of these

8. *Let* $\sum a_n$ be a convergent series converging to the sum *s*. Then $\lim_{x\to\infty} a_n$ = _____.

a) 0 b) 1 c) 2 d) 3

9. A series whose terms are alternatively positive and negative is called an _____series.

a) convergent b) divergent c) oscillating d) alternating

10. A series $\sum a_n$ is said to be _____ if the series $\sum |a_n|$ is convergent.

a) convergent b) divergent

c) absolutely convergent d) none of these

SECTION – B

Answer ALL Questions : **(5 × 7 = 35)**

11. a) State and prove Cauchy – Schwarz inequality.

(OR)

b) State and prove Weierstrass' inequalities.

12. a) Prove that any convergent sequence is a bounded sequence.

(OR)

b) Prove that a sequence cannot converge to two different limits.

13. a) Prove that any Cauchy sequence is a bounded sequence.

(OR)

b) Prove that any convergent sequence is a Cauchy sequence.

14. a) Apply Cauchy's general principle of convergence to show that the series $\sum \frac{1}{n}$ is not convergent.

(OR)

b) Discuss the convergence of the series $\sum \frac{1}{\sqrt{n^3+1}}$.

15. a) Show that the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots\ldots$ converges.

(OR)

b) Show that the series $\sum \frac{x^{n-1}}{(n-1)!}$ Converges absolutely for all values of *x*.

SECTION – C

Answer any THREE Questions : **(3 × 10 = 30)**

16. State and prove Triangle inequalities.

17. Prove that

a) A monotonic increasing sequence which is bounded above converges to its l.u.b.

b) A monotonic increasing sequence which is not bounded above diverges to ∞.

18. State and prove Cauchy's first limit theorem.

19. Discuss the convergence of the series $\sum \frac{\sqrt{n+1}-\sqrt{n}}{n^p}$.

20. Prove that any absolutely convergent series is convergent.

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B.Sc. Mathematics Degree (Semester) Examinations, April 2017
Part – III : Core Subject : Fourth Semester : Paper – II

DYNAMICS
Under CBCS – Credit 4

Time: **3** Hours Max. Marks: **75**

SECTION – A

Answer ALL Questions : **(10 × 1 = 10)**

1. The Horizontal range R is _____.
   a) $\frac{v^2 \sin^2 \alpha}{2g}$ b) $\frac{v \sin \alpha}{g}$ c) $\frac{2v \sin \alpha}{g}$ d) $\frac{v^2 \sin 2\alpha}{g}$
2. If the sphere is perfectly elastic $e = 1$ and the loss of KE is _____.
   a) 0 b) 1 c) constant d) $u = v$
3. In SHM, the periodic time is
   a) $\frac{\pi}{\sqrt{\mu}}$ b) $\frac{2\pi}{\sqrt{\mu}}$ c) $\frac{\mu}{\pi}$ d) $\frac{\mu}{2\pi}$
4. The polar equation of the equiangular spiral is
   a) $r = e^{\theta}$ b) $r = ae^{\sin\theta}$ c) $r = ae^{\theta \cot \alpha}$ d) $r = a \cot \alpha$
5. In M.I., the formula for uniform circular ring about a diameter is
   a) $\frac{2Ma^2}{3}$ b) $\frac{Ma^2}{3}$ c) $\frac{Ma^2}{4}$ d) $\frac{Ma^2}{2}$
6. The time of flight of a projectile is _____.
7. If two equal perfectly elastic spheres impinge, they interchange their velocities in the direction of the _____.
8. Periodic time of a simple pendulum is _____.
9. Differential equation of a central orbit is _____.
10. The parallel axes theorem is true for any _____.

SECTION – B

Answer ALL Questions : **(5 × 7 = 35)**

11. a) A stone is thrown with a velocity of $39.2\,{}^{m}\!/_{\text{sec}}$ *at* 30^{o} to the horizontal. Find at what times it will be at a height of $14.7\,m$.

(OR)

b) *If* $\hbar$ *and* $\hbar'$ be the greatest heights in the two paths of a projectile with a given velocity for a given range R. Prove that $R = 4\sqrt{\hbar\hbar'}$.

12. a) Define the following i) Impulse

ii) Impinge directly and Impinge obliquely

(OR)

b) Obtain the Loss of $K.E.$ due to direct impact of two smooth spheres.

13. a) A particle is moving with SHM and while making an oscilation frome one extreme position to other, its distances from the centre of oscillation at three consecutive seconds are x_1, x_2, x_3. Prove that the period of oscillation is $\dfrac{2\pi}{\cos^{-1}\left(\dfrac{x_1 + x_3}{2x_2}\right)}$.

(OR)

b) Obtain the differential equation of the SHM and solve them.

14. a) Derive the differential equation of central orbits in polar coordinates.

(OR)

b) Find the law of force to an internal point under which a body will describe a circle.

15. a) State and prove the parallel axes theorem.

(OR)

b) Find the moment of inertia (M.I.) of uniform circular disc.

SECTION – C

Answer any THREE Questions : **(3 × 10 = 30)**

16. Find the range on an inclined plane of a projectile.

17. Obtain the Loss of KE due to oblique impact of two smooth spheres.

18. Find the composition of two SHM's of the same period in two per pendicular directions.

19. Find the law of force towards the pole under which the curve $r^n = a^n \cos n\theta$ can be described.

20. State and prove the perpendicular axes theorem.

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B.Sc. Mathematics Degree (Semester) Examinations, April 2017
Part – III : Core Subject : Sixth Semester : Paper – I

LINEAR ALGEBRA
Under CBCS – Credit 4

Time: **3** Hours Max. Marks: **75**

SECTION – A

Answer ALL Questions : **(10 × 1 = 10)**

1. In a linear transformation $T:V\to W$ is a homomorphism *if* T is $1-1$ then T is called
   a) monomorphism b) epimorphism c) isomorphism d) None
2. Schwartz's inequality is $|\langle x,y\rangle| \le$ _____.
   a) $\|x\|+\|y\|$ b) $\|x\|\,\|y\|$ c) $\|x\|-\|y\|$ d) $\|x\|/\|y\|$
3. *If* A is 3×4 matrix and B is 4×2 matrix, then product AB is ____.
   a) 2×3 b) 3×3 c) 3×2 d) 4×4
4. *If* λ is an eigen value of a non singular matrix A then $1/\lambda$ is an eigen value of _____.
   a) A^{-1} b) A^{T} c) A^{*} d) None
5. $L(V,V,F)$ is a vector space of dimension _____.
   a) $n+1$ b) n^2 c) $n-1$ d) n
6. In a vector space $(\alpha+\beta)u =$ _____ for all $u\in V$ *and* $\alpha,\beta\in F$.
7. The norm of x is denoted by $\|x\| =$ _____.
8. A square matrix A is skew symmetric iff _____.
9. Any square matrix A satisfies its _____.
10. *Let* V be a vector space over R then an inner product on V is a _____ form on V.

SECTION – B

Answer ALL Questions : **(5 × 7 = 35)**

11. a) Prove that the intersection of two subspaces of a vector space is a subspace.

(OR)

b) *Let* $T:V\to W$ be a linear transformation then prove that $\dim V = rank\ T + nullity\ T$.

12. a) *Let* V be a vector space of polynomial with inner product defined by $\langle f,g\rangle = \int_0^1 f(t)g(t)dt$. *Let* $f(t)=t+2$ *and* $g(t)=t^2-2t-3$, find $|\langle f,g\rangle|$.

(OR)

b) State and prove the triangle inequality.

13. a) *Let* A *and* B be two $m\times n$ matrices then prove that $(A+B)^T = A^T + B^T$.

(OR)

b) Find the rank of the matrix $A=\begin{pmatrix}4&2&1&3\\6&3&4&7\\2&1&0&7\end{pmatrix}$.

14. a) Find the characteristic equation of $A=\begin{pmatrix}1&0&2\\0&1&2\\1&2&0\end{pmatrix}$.

(OR)

b) Find the sum and product of the eigen values of the matrix $A=\begin{pmatrix}3&-4&4\\1&-2&4\\1&-1&3\end{pmatrix}$ without actually finding the eigen values.

15. a) Obtain the matrix representing the linear transformation $T:V_3(R)\to V_3(R)$ given by $T(a,b,c)=(3a,a-b,2a+b+c)$ with respect to the standard basis $\{e_1,e_2,e_3\}$.

(OR)

b) *Let* f be a symmetric bilinear form defined on V. *Let* q be the associated quadratic form then prove that $f(u,v)=\{q(u+v)-q(u-v)\}/4$.

SECTION – C

Answer any THREE Questions : **(3 × 10 = 30)**

16. Prove that any vector space of dimension n over a field F is isomorphic to $V_n(F)$.

17. *Let* V be a finite dimensional inner product space. *Let* W be a subspace of V. Then prove that V is the direct sum of W *and* $W^{\perp}$.

18. Compute the inverse of the matrix $A=\begin{pmatrix}2&-1&1\\-15&6&-5\\5&-2&2\end{pmatrix}$ using adjoint of matrix A.

19. Verify Cayley Hamilton theorem for the matrix Find the eigen values and eigen vectors of the matrix $A=\begin{pmatrix}1&2\\4&3\end{pmatrix}$.

20. *Let* V be a vector space over a field F. Then prove that $L(V,V,F)$ is a vector space over F under addition and multiplication defined by $(f+g)(u,v)=f(u,v)+g(u,v)$ *and* $(\alpha f)(u,v)=\alpha\ f(u,v)$.

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B.Sc. Mathematics Degree (Semester) Examinations, April 2017
Part – III : Core Subject : Sixth Semester : Paper – II

OPERATIONS RESEARCH
Under CBCS – Credit 5

Time: **3** Hours Max. Marks: **75**

SECTION – A

Answer ALL Questions : **(10 × 1 = 10)**

1. In a EOQ problem minimum cost equation is
   a) $\sqrt{2Dc_1c_S}$ b) $\sqrt{2D\,c_1/c_S}$ c) $\sqrt{2D\,c_S/c_1}$ d) $\sqrt{2c_1c_S}$
2. Average number of customers arriving per unit of time is
   a) μ b) λ c) ρ d) $1/\rho$
3. _____ is the shortest possible time to complete the activity if all goes well.
   a) most likely time b) pessimistic time
   c) optimistic time d) Normal time
4. _____ is the time between starting the first job and completing the last one.
   a) Total elapsed time b) idle time
   c) processing time d) Normal time
5. _____ types of replacement policy.
   a) 2 b) 3 c) 4 d) 1
6. The time between placing an order and its actual arrival in the inventory is known as _____.
7. Poisson probability law with mean λt is _____.
8. The longest time path through a network is called the _____.
9. _____ means that passing is not allowed.
10. The total cost T incurred on the machine during n years is _____.

SECTION – B

Answer ALL Questions : **(5 × 7 = 35)**

11. a) Describe the following terms related to the inventory problems.
    i) Set-up cost ii) Shortage cost
    iii) ordering cost iv) production cost **(OR)**
    b) An oil engine manufacturer purchases lubricants at the rate of Rs.42 per piece from a vendor. The requirement of these lubricants is 1800 per year. What should be the order quantity per order, if the cost per placement of an order is Rs. 16 and inventory carrying charges per rupee per year is only 20 paise.

12. a) Write the characteristics of a queueing system. **(OR)**
    b) Find the average number of customers in the queuing system for $(M/M/1):(\infty/F/FO)$ model.

13. a) Differentiate between PERT and CPM. **(OR)**
    b) Construct the network activity for the following conditions are satisfied. $B<E,F;\ C<G,L;\ E,G<H;\ L,H<I;\ L<M;$ $H,M<N;\ H<J;\ I,J<P;\ P<Q.$ The notation $X<Y$ means that the activity X must be finished before Y can begin.

14. a) What are the assumptions underlying a sequencing problem.
    (OR)
    b) We have seven jobs each of which has to go through the machine M_1 *and* M_2 in the order M_1, M_2. The processing timings (in hours) for the jobs are given below. Determine a sequence of these jobs that will minimize the total elapsed time T.

Job	1	2	3	4	5	6	7
Machine M_1	3	12	15	6	10	11	9
Machine M_2	8	10	10	6	12	1	3

15. a) Derive the average annual total cost when 't' is continuous variable.
    (OR)
    b) What is replacement problem?
    What are the types of replacement problems?

SECTION – C

Answer any THREE Questions : **(3 × 10 = 30)**

16. Derive the economic lot size formula when the demand is uniform and each production run.

17. A T.V. repairman finds that the time spent on his jobs has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they came in, and if the arrival of sets is approximately Poisson with an average rate of 10 per 8-hour day, what is repairman's expected idle time each day? How many jobs are ahead of the average set just brought in?

18. The following table shows the jobs of a network along with their time (in days) estimates

Jobs:	1–2	1–3	1–4	2–5	2–6	3–6	4–7	5–7	6–7
Optimistic Time:	5	18	26	16	15	6	7	7	3
Pessimistic Time:	10	22	40	20	25	12	12	9	5
Most likely Time:	8	20	33	18	20	9	10	8	4

Draw the project network and determine the task times and their variances and also the critical path.

19. Determine the optimal sequence of jobs that minimizes the total elapsed time based on the following information processing time on machines is given in hours and passing is not allowed.

Job	A	B	C	D	E	F	G
Machine A	3	8	7	4	9	8	7
Machine B	4	3	2	5	1	4	3
Machine C	6	7	5	11	5	6	12

20. A truck owner finds from his past records that the maintenance costs per year, of a truck whose purchase price is Rs. 8000 are as given below.

Year :	1	2	3	4	5	6	7	8
Maintenance cost :	1000	1300	1700	2200	2900	3800	4800	6000
Resale price :	4000	2000	1200	600	500	400	400	400

Determine at which time it is profitable to replace the truck.

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B.Sc. Mathematics Degree (Semester) Examinations, April 2017
Part – III : Elective Subject : Sixth Semester : Paper – I

GRAPH THEORY
Under CBCS – Credit 5

Time: **3** Hours Max. Marks: **75**

SECTION – A

Answer ALL Questions : **(10 × 1 = 10)**

1. A point v of degree 1 is called on _____.
   a) isolated point b) even c) odd d) end point
2. Ramsey numbers $r(3,3) =$ _____.
   a) 1 b) 2 c) 3 d) 4
3. The partition $(5,5,5,5,5,5)$ is _____.
   a) graphical b) not graphical c) isomorphic d) not isomorphic
4. A single point is considered as a walk of length _____.
   a) zero b) one c) odd degree d) trail
5. A graph is polyhedral iff it is _____.
   a) planar b) non planar c) connected d) not connected
6. Every Hamiltonian graph is _____.
7. Every Planar graph has genus _____.
8. The Petersen graph has chromatic index _____.
9. The constant term in $f(G,\lambda)$ is _____.
10. A _____ has atmost one perfect matching.

SECTION – B

Answer ALL Questions : **(5 × 7 = 35)**

11. a) Prove that $\delta \le \frac{2q}{p} \le \Delta$.

(OR)

b) Prove that $\overline{|(G)} = |(\overline{G})$.

12. a) Prove that a graph G with P points and $\delta \ge \frac{p-1}{2}$ is connected.

(OR)

b) Prove that for any graph G, $k \le \lambda \le \delta$.

13. a) Prove that every connected graph has a spanning tree.

(OR)

b) Prove that a graph is Hamiltonian $\Leftrightarrow$ its closure is Hamiltonion.

14. a) Find the number of perfect matchings in the complete graph K_{2n}.

(OR)

b) State and prove Euler's polyhedron formula.

15. a) Prove that for any graph G, $\chi \le \Delta + 1$.

(OR)

b) *If* G is a tree with n points, $n \ge 2$ then prove that $f(\overline{G},\lambda) = \lambda(\lambda-1)^{n-1}$.

SECTION – C

Answer any THREE Questions : **(3 × 10 = 30)**

16. Prove that *a set* $s \subseteq v$ is an independent set of $G \Leftrightarrow v - s$ is a covering of G. Also prove that $\alpha + \beta = p$.
17. Prove that a graph G with atleast two points is bipartite $\Leftrightarrow$ all its cycles are of even length.
18. Prove that the Petersen graph is Non – Hamiltonian.
19. State and prove Hall's Marriage theorem.
20. Prove that every planar graph is 5 – colourable.

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B.Sc. Mathematics Degree (Semester) Examinations, April 2017
Part – III : Elective Subject : Sixth Semester : Paper – II

COMPLEX ANALYSIS
Under CBCS – Credit 5

Time: **3** Hours Max. Marks: **75**

SECTION – A

Answer ALL Questions : **(10 × 1 = 10)**

1. *Let* T be a bilinear transformation with two fixed points α, β. *If* k real, then T is called _____.
   a) hyperbolic b) elliptic c) curve d) none of these
2. *Let* T be a bilinear transformation with two fixed points α, β. *If* $|k| = 1$, then T is called _____.
   a) hyperbolic b) elliptic c) curve d) none of these
3. CR equations in complex form is _____.
   a) $f_x = if_y$ b) $f_x = f_y$ c) $f_x = -if_y$ d) none of these
4. A function f defined in a region D of the complex plane is said to be _____ at a point a ε D *if* f is differentiable at every point of some neighbourhood of a.
   a) analytic b) elliptic c) hyperbolic d) connected
5. Say True or False: $\left|\int_a^b f(t)\,dt\right| \le \int_a^b |f(t)|\,dt.$
6. A region D is said to be _____ if every simple closed curve lying in D encloses only points of D.
   a) analytic b) simply connected
   c) not connected d) none of these

7. There are _____ types of singularities.

a) one b) two c) three d) four

8. *Let* a be an isolated singularity for $f(z)$. Then a is called a _____ if the principal part of $f(z)$ *at* $z=a$ has no terms.

a) removable singularity b) pole

c) essential singularity d) none of these

9. A polynomial of degree n with complex coefficients has _____ zeros in C.

a) n b) $n+1$ c) $n-1$ d) none of these

10. *Let* a be an isolated singularity for $f(z)$. Then the _____ of $f(z)$ at a is defined to be the coefficient of $\frac{1}{z-a}$ in the Laurent's series expansion of f(z) about a.

a) pole b) residue c) singular point d) none of these

SECTION – B

Answer ALL Questions : **(5 × 7 = 35)**

11. a) Find the image of the circle $|z-3i|=3$ under the map $w=\frac{1}{z}$.

(OR)

b) Show that $w=\frac{z-1}{z+1}$ maps the imaginary axis in the $z-plane$ onto the circle $|w|=1$.

12. a) Prove that the functions $f(z)$ *and* $\overline{f(\overleftarrow{z})}$ are simultaneously analytic.

(OR)

b) Find constants a and b so that the function $f(z)=a(x^2-y^2)+ibxy+c$ is differentiable at every point

13. a) State and prove maximum modulus theorem.

(OR)

b) State and prove Cauchy's inequality.

14. a) State and prove Riemann's theorem.

(OR)

b) State and prove Weierstrass theorem for essential singularity.

15. a) State and prove Argument principle.

(OR)

b) Find the residue of $\frac{1}{(z^2+a^2)^2}$ *at* $z=ai$.

SECTION – C

Answer any THREE Questions : **(3 × 10 = 30)**

16. Find the bilinear transformation which maps the points $0,-i,-1$ into $i,1,0$ respectively.

17. Prove that $u=2x-x^3+3xy^2$ is harmonic and find its harmonic conjugate. Also find the corresponding analytic function.

18. State and prove Cauchy's integral formula.

19. State and prove Taylor's theorem.

20. Evaluate $\int_0^{2\pi}\frac{d\theta}{5+4\sin\theta}$.

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B.A. / B.Sc. Degree (Semester) Examinations, April 2017
Part – IV : Non-Major Elective Subject : Second Semester : Paper–I

STATISTICS AND OPERATIONS RESEARCH
Under CBCS – Credit 2

Time: **2** Hours Max. Marks: **75**

SECTION – A

Answer ALL Questions : **(10 × 1 = 10)**

1. The arithmetic mean of 1, 2, 3, 4, 5 is _____.
2. *Let* $x_1, x_2, \ldots\ldots\ldots\ldots, x_n$ be n distinct values of a variate with corresponding frequencies $f_1, f_2, \ldots\ldots\ldots\ldots, f_n$, then $\bar{x} =$ _____.
3. The median of 66, 65, 64, 70, 61, 60, 56 is _____.
4. The mode of 7, 5, 6, 7, 9, 7, 1 is _____.
5. If mean = 6, mode = 3, then median is _____.
6. The range of 7, 5, 3, 2, 14, 15 is _____.
7. A linear programming problem involving 2 decision variables can easily solved by graphical method. **(True / False)**
8. If anLPP is subject to a set of linear equations, then these equations are called _____.
9. The square of the standard deviation is called _____.
10. The Assignment problem is a special case of _____.

SECTION – B

Answer ALL Questions : **(4 × 10 = 40)**

11. a) Calculate the arithmetic mean from the following frequency table.

Weight in Kgs	50	48	46	44	42	40
No.of persons	12	14	16	13	11	09

(OR)

b) Find the missing frequency from the following table, given that their arithmetic mean is 7.3.

X	2	6	7	8	9
f	4	6	12	?	8

12. a) Find the median for the following data.

X	1	2	3	4	5	6	7	8	9
F	8	10	11	16	20	25	15	9	6

(OR)

b) Find the mode for the following data.

Size of shoes	3	4	5	6	7	8	9	10
No.of persons	10	28	38	42	45	15	8	7

13. a) Find the median and quartile marks of 10 students in statistics test given by 40, 90, 61, 68, 72, 43, 50, 84, 75, 33.

(OR)

b) Find Q_1 and Q_3 for the following distribution.

Marks	**5 – 10**	**10 – 15**	**15 – 20**	**20 – 25**	**25 – 30**	**30 – 35**	**35 – 40**	**40 – 45**
Students	5	6	15	10	5	4	2	2

14. a) Obtain the initial basic feasible solution for the following transportation problem using North West Corner rule.

	D	**E**	**F**	**G**	**Available**
A	19	30	50	10	7
B	70	30	40	60	9
C	40	08	70	20	18
Demand	5	8	7	14	

(OR)

b) Find the feasible solution for the LPP, using graphical method.

$$\max \ z = 3x_1 + 5x_2,$$

Subject to the constraints

$$x_1 + 2x_2 \le 2000,$$

$$x_1 + x_2 \le 1500,$$

$$x_2 \le 600 \ \ and \ \ x_1 \ge 0, \ \ x_2 \ge 0.$$

SECTION – C

Answer any TWO Questions : **(2 × 12½ = 25)**

15. Find the standard deviation and coefficient of variation for the following frequency table.

Height	**60 – 62**	**63 – 65**	**66 – 68**	**69 – 71**	**72 – 74**
No.of students	5	18	42	27	8

16. Obtain the initial basic feasible solution for the following transportation problem.

	I	II	III	IV	Available
A	7	3	5	5	34
B	5	5	7	6	15
C	8	6	6	5	12
D	6	1	6	4	19
Demand	21	25	17	17	

17. Find the standard deviation and quartile deviation for the following frequency table.

Marks	**30 – 39**	**40 – 49**	**50 – 59**	**60 – 69**	**70 – 79**	**80 – 89**
Students	37	50	42	21	11	03

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B.Sc. Mathematics Degree (Semester) Examinations, April 2017
Part – IV : Skill Based Subject : Fourth Semester : Paper – I

COMPETITIVE MATHEMATICS
Under CBCS – Credit 2

Time: **2** Hours Max. Marks: **75**

SECTION – A

Answer ALL Questions : **(10 × 1 = 10)**

1. Find the highest common factor of 36 *and* 84
   a) 4 b) 6 c) 12 d) 18
2. Which of the following is a pair of co-prime?
   a) $(16,\ 62)$ b) $(18,\ 25)$ c) $(21,\ 35)$ d) $(23,\ 92)$
3. The value of $\sqrt{0.121}$ is
   a) 0.011 b) 0.11 c) 0.347 d) 1.1
4. The average of the first 40 natural numbers is
   a) 20.5 b) 21.5 c) 22.5 d) 23.5
5. Which of the following fractions is the smallest?
   a) $\frac{13}{16}$ b) $\frac{15}{19}$ c) $\frac{17}{21}$ d) $\frac{7}{8}$
6. Mohan is younger than Gowtham by 4 years. If their ages are in the respective ratio of $7:9$ how old is Mohan?
   a) 16 years b) 18 years c) 28 years d) None of these
7. Write down the formula for gain % _____.
8. *If* $\frac{A}{3}=\frac{B}{4}=\frac{C}{5}$ then $A:B:C$ is
   a) $4:3:5$ b) $5:4:3$ c) $3:4:5$ d) $20:15$
9. 2 is what percent of 50?
   a) 20% b) 4% c) 6% d) 8%
10. *If* $x=1+\sqrt{2}$ *and* $y=1-\sqrt{2},$ the value of $\left(x^2+y^2\right)$ is
    a) 6 b) 12 c) 4 d) 2

SECTION – B

Answer ALL Questions : **(4 × 10 = 40)**

11. a) i) Find the H.C.F of 108, 288 *and* 360.

ii) Arrange the fractions $\frac{17}{18}, \frac{31}{36}, \frac{43}{45}, \frac{59}{60}$ in the ascending order.

(OR)

b) i) If $\sqrt{2} = 1.4142$, Find the value of $\left(\frac{\sqrt{2}}{2+\sqrt{2}}\right)$.

ii) Evaluate $\sqrt{10+\sqrt{25+\sqrt{108+\sqrt{154+\sqrt{225}}}}}$.

12. a) i) The average of four consecutive even numbers is 25. Find the largest of these numbers.

ii) The product of the ages of Ramesh and Jothi is 240. If twice the age of Jothi is more than Ramesh age by 4 years, what is Jothi's age?

(OR)

b) i) When the price of a product was decreased by 10% the numbers sold increased by 30%. What was the effect on the total revenue?

ii) A person incurs 5% loss by selling a watch for Rs.1140. At what price should the watch be sold to earn 5% profit?

13. a) i) If $x : y = 3 : 4$, find $(4x+5y):(5x-2y)$.

ii) Simplify : $\frac{0.05\times0.05\times0.05+0.04\times0.04\times0.04}{0.05\times0.05-0.05\times0.04+0.04\times0.04}$.

(OR)

b) i) Find the average of all prime numbers between 30 *and* 50.

ii) Divide Rs.1162 among A, B, C in the ratio $35 : 28 : 20$.

14. a) i) Find the largest number of four digits exactly divisible by 12,15,18 *and* 27.

ii) If 40% of a number is equal to two-thirds of another number, what is the ratio of the first number to the second number?

(OR)

b) i) P and Q started a business investing Rs.85,000 and Rs.15,000 respectively. In what ratio the profit earned after 2 years be divided between P and Q respectively.

ii) Find the value of $\sqrt{3}$ upto three places of decimal.

SECTION – C

Answer any TWO Questions : **(2 × 12½ = 25)**

15. a) Find the L.C.M. of 22, 54, 108, 135 *and* 198.

b) Find the value of $35.7 - \left[3+\frac{1}{3+\frac{1}{3}}\right] - \left[2+\frac{1}{2+\frac{1}{2}}\right]$.

16. a) If $a = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $b = \frac{\sqrt{5}-1}{\sqrt{5}+1}$, Find the value of $\left[\frac{a^2+ab+b^2}{a^2-ab+b^2}\right]$.

b) An article was sold for Rs.144. If the percentage of profit was numerically equal to the cost price, how much is the cost of the article.

17. a) Find i) The Fourth Proportional to 4, 9, 12.

ii) The Third Proportional to 16 and 36.

iii) The Mean Proportional between 0.08 and 0.18.

b) A, B *and* C enter into partnership. A invests 5 times as much as B invests *and* B invests two- thirds of what C invests. At the end of the year, the profit earned is Rs.6,000. What is the share of B?

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B.Sc. Mathematics Degree (Semester) Examinations, April 2017
Part – IV : Skill Based Subject : Sixth Semester : Paper – I

APPLIED STATISTICS
Under CBCS – Credit 2

Time: **2** Hours Max. Marks: **75**

SECTION – A

Answer ALL Questions : **(10 × 1 = 10)**

1. The qualitative characterstics of a population are called _____.
   a) Frequencies b) Attributes c) Positive class d) None
2. Class frequencies of the type $(A),(B),(ABC)$..... are known as _____ class frequencies.
   a) positive b) negative c) contrary d) None
3. For any four given attributes, the total number of positive class frequencies is _____.
   a) 15 b) 81 c) 17 d) 16
4. For any three given attributes, the total numbers of class frequencies is _____.
   a) 3^3 b) 2^3 c) 3^2 d) 7
5. The classes of highest order is called _____.
6. If $(AB) \neq \frac{(A)(B)}{N}$, we say that A and B are _____.
7. If the positive class frequencies are denoted by A,B,C,D then the negative class frequencies are _____.
8. A and B are positively associated if $\delta > 0$. (True / False)
9. The relation between the Yule's coefficient Q and the coefficients of colligation y is _____.
   a) $Q = \frac{1+y^2}{2y}$ b) $Q = \frac{2y}{1+y^2}$ c) $Q = \frac{y}{1-y^2}$ d) None
10. Write down the formula for Yule's coefficient of association.

SECTION – B

Answer ALL Questions : **(4 × 10 = 40)**

11. a) i) Given that $(A)=(\alpha)=(B)=(\beta)=\frac{N}{2}$ show that

i) $(AB)=(\alpha\beta)$ ii) $(A\beta)=(\alpha B)$

ii) Examine the consistency of data :

$(A)=800,\quad (B)=700,\quad (AB)=660,\quad N=1000$

(OR)

b) Given $(A)=30\quad (B)=25;\quad (\alpha)=30\quad (\alpha\beta)=20.$ Find

(i) N (ii) β (iii) (AB) (iv) $A\beta$ (v) αB

12. a) In a very hartly fought battle 70% of the soliders atleast lost an eye, 75% atleast lost an ear, 80% atleast an arm and 85% atleast lost a leg. How many atleast must have lost all the four?

(OR)

b) Find the limits of (BC) for the following available data. $N=125,$

$(A)=48\quad (B)=62\quad (C)=45\quad (A\beta)=7\ \ \&\ \ (A\gamma)=18.$

13. a) Find the greatest and the least values of (ABC) *If* $(A)=50,$

$(B)=60,\quad (C)=80,\quad (AB)=35,\quad (AC)=45$ *and* $(BC)=42.$

(OR)

b) Check whether the attributes A *and* B are independent, given that

i) $(A)=30,\quad (B)=60,\quad (AB)=12,\quad N=150.$

ii) $(AB)=256\quad (\alpha B)=768\quad (A\beta)=48\quad (\alpha\beta)=144.$

14. a) From the following data compare the association between the marks in Physics and Chemistry in M.K.U. and M.S.U.

University	M.S.U.	M.K.U.
Total number of candidates	200	1600
Pass in Physics	80	320
Pass in Chemistry	40	90
Pass in Physics & Chemistry	20	30

(OR)

b) Of 500 men in a locality exposed to cholera 172 in all were attacked. 178 were inoculated and of these 128 were attacked. Find the numbers of persons,

i) not inoculated, not attacked

ii) inoculated, not attacked

iii) not inoculated, attacked

SECTION – C

Answer any TWO Questions : **(2 × 12½ = 25)**

15. Given the following positive class frequencies. Find the remaining class frequencies. $(N)=20,(A)=9,\ (B)12,\ (C)=8$

$(AB)=6,\ (BC)=4,\ (CA)=4,\ (ABC)=3.$

16. The following is the characteristics showing the lives in hours of four batches of electric bulbs sold in different shops. Perform an analysis of variance and state your conclusion.

Batches	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
A	1600	1610	1650	1680	1700	1720	1800	---
B	1580	1640	1640	1700	1750	---	---	---
C	1460	1550	1600	1620	1640	1660	1740	1820
D	1510	1520	1530	1570	1600	1680	---	---

17. Analyse the variance in the following Latin Square.

A8	C18	B9
C9	B18	A16
B11	A10	C20

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B.Sc. Mathematics Degree (Semester) Examinations, April 2017
Part – IV : Skill Based Subject : Sixth Semester : Paper – II

BOOLEAN ALGEBRA
Under CBCS – Credit 2

Time: **2** Hours Max. Marks: **75**

SECTION – A

Answer ALL Questions : **(10 × 1 = 10)**

1. In the set of all integers $\mathbb{Z}$, we define a relation ρ by $a \rho b \Leftrightarrow a < b$. Then ρ is a
   a) Reflexive relation b) Symmetric relation
   c) Transitive relation d) None of the above
2. Define :- Equivalence relation.
3. Define :- Comparable elements in a poset.
4. A poset in which any two elements are comparable is called a _____.
5. If a relation ρ defined on a set A satisfies the condition "$a \rho b$, $b \rho a \Rightarrow a = b$", then ρ is called a _____.
   a) Symmetric relation b) Anti – symmetric relation
   c) Reflexive relation d) Transitive relation
6. The poset $(\mathbb{Z}, \leq)$ has neither the least element nor the greatest element. (True / False)
7. In the lattice $(\mathbb{N}, \leq)$, $a \wedge b =$ _____.
8. M_5 is a distributive lattice. (True / False)
9. *Let* L be a lattice with 0 *and* 1. *Let* $a \in L$. An element $a' \in L$ is called a complement of 'a' *if* _____.
10. State De Morgan's laws in a Boolean algebra.

SECTION – B

Answer ALL Questions : **(4 × 10 = 40)**

11. a) i) If ρ and σ are two equivalence relations defined on a set S, then prove that $\rho \cap \sigma$ is also an equivalence relation.

ii) In $\mathbb{Z}$, define a relation ρ by "$a \ \rho \ b \Leftrightarrow ab$ is a perfect square". Show that ρ is an equivalence relation in $\mathbb{Z}$.

(OR)

b) Give an example of a relation which is

i) reflexive, symmetric and transitive

ii) reflexive, symmetric and not transitive

iii) reflexive, not symmetric and transitive

iv) not reflexive, symmetric and transitive

v) not reflexive, not symmetric and transitive

12. a) Draw a diagram for the following posets.

i) $(\{10,9,8,6,5\}, \leq)$.

ii) $(A, \leq)$ where $A = \{2,5,8,20,40\}$, $a \leq b \Leftrightarrow a$ *divides* b.

iii) M_5.

iv) $(\wp(\{1,2\}), \subseteq)$.

v) (The set of all sub groups of S_3, $\subseteq$).

(OR)

b) Define the following and give examples :

i) Upper bound

ii) Least upper bound

iii) Lower bound

iv) Greatest lower bound

v) Lattice

13. a) In a lattice $(L, \leq)$, state and prove the

i) idempotent laws

ii) commutative laws

iii) associative laws

iv) absorption laws

(OR)

b) Show that in any lattice, $L_5 : a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$ *and* $L_5' : a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$ are equivalent.

14. a) i) Define a Boolean Algebra. Show that for any set S, $(P(S), \subseteq)$ is a Boolean algebra

ii) Prove that a Boolean algebra can not have exactly three elements.

(OR)

b) i) Define: Modular lattice.

ii) Prove that the lattice of normal sub groups of any group is a modular lattice.

SECTION – C

Answer any TWO Questions : **(2 × 12½ = 25)**

15. a) Let ρ be an equivalence relation defined on a set S. Then prove that the following are equivalent.

i) $a \ \rho \ b \Leftrightarrow [a] = [b]$.

ii) Any two distinct equivalence classes are disjoint.

iii) S is the union of all equivalence classes.

b) Show that the union of two equivalence relations need not be an equivalence relation.

16. a) In any distributive lattice L, show that

$$(a \vee b) \wedge (b \vee c) \wedge (c \vee a) = (a \wedge b) \vee (b \wedge c) \vee (c \wedge a), \forall a, b, c \in L.$$

b) Show that N_5 is not a distributive lattice.

17. a) Define a lattice.

b) *Let* L be a lattice and $a, b \in L$. Then prove that the following are equivalent.

i) $a \leq b$ ii) $a \vee b = b$ iii) $a \wedge b = a$

c) *Let* L be the set of all sub groups of a group G.

In L we define $A \leq B \Leftrightarrow A \subseteq B$. Then prove that L is a lattice.

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